核物理概论

第一章：原子核整体性质

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中山大学物理与天文学院

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课程介绍

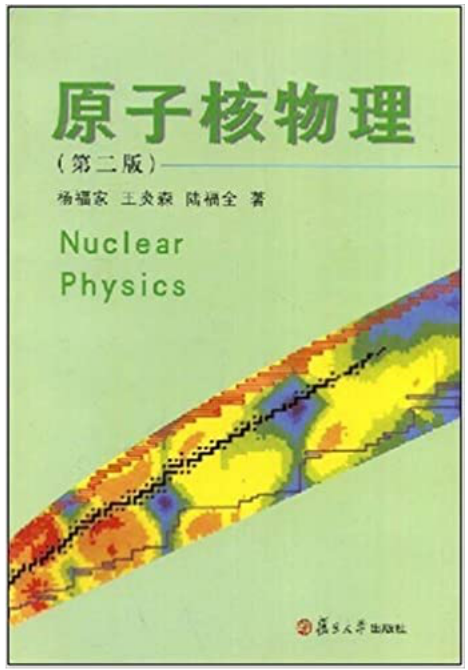
- 参考书籍

1、主讲教材 **（主讲教材尽量使用“马工程”和国家规划教材）**

《原子核物理》 杨福家、王炎森、陆福全，复旦大学出版社，2002 年

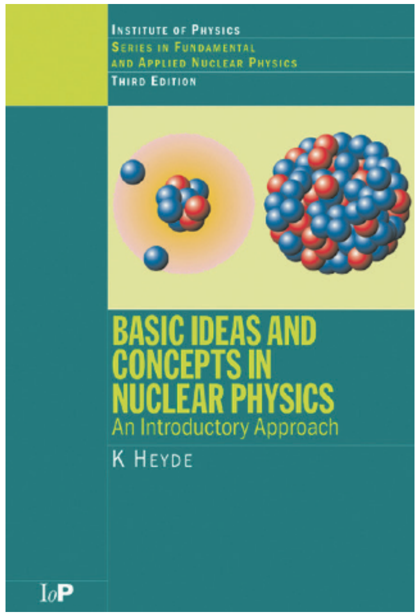
2、辅助教材

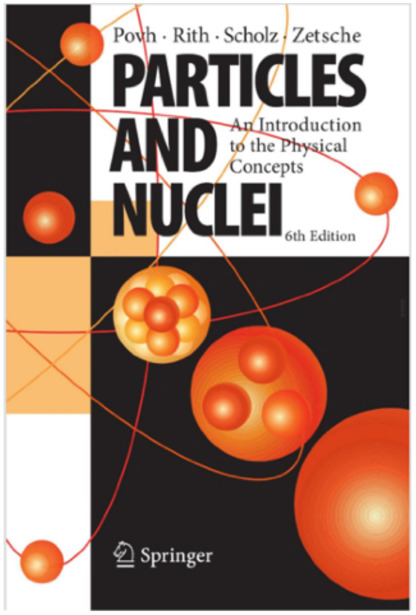
《原子核物理》 卢希庭，原子能出版社，2002 年

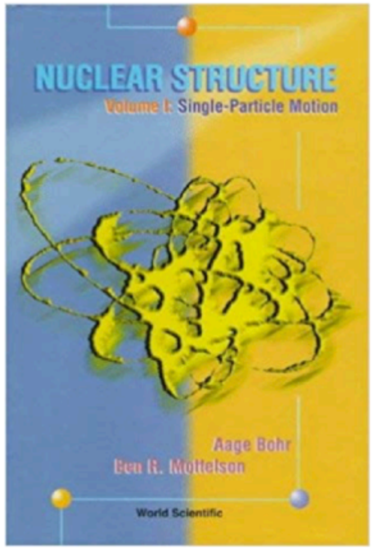


- Books

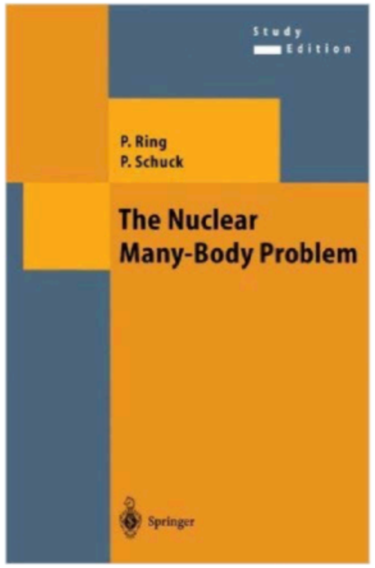
1. **《Particles and Nuclei: An Introduction to the Physical Concepts》, Bogdan Povh, Klaus Rith, Christoph Scholz, Frank Zetsche, Werner Rodejohann, Springer-Verlag Berlin Heidelberg, 2008**
2. **《Basic ideas and concepts in nuclear physics: An introductory approach》, K. Heyde, CRC Press; 3rd edition, 2004**







Bohr-Mottelson



Ring-Schuck

课程介绍

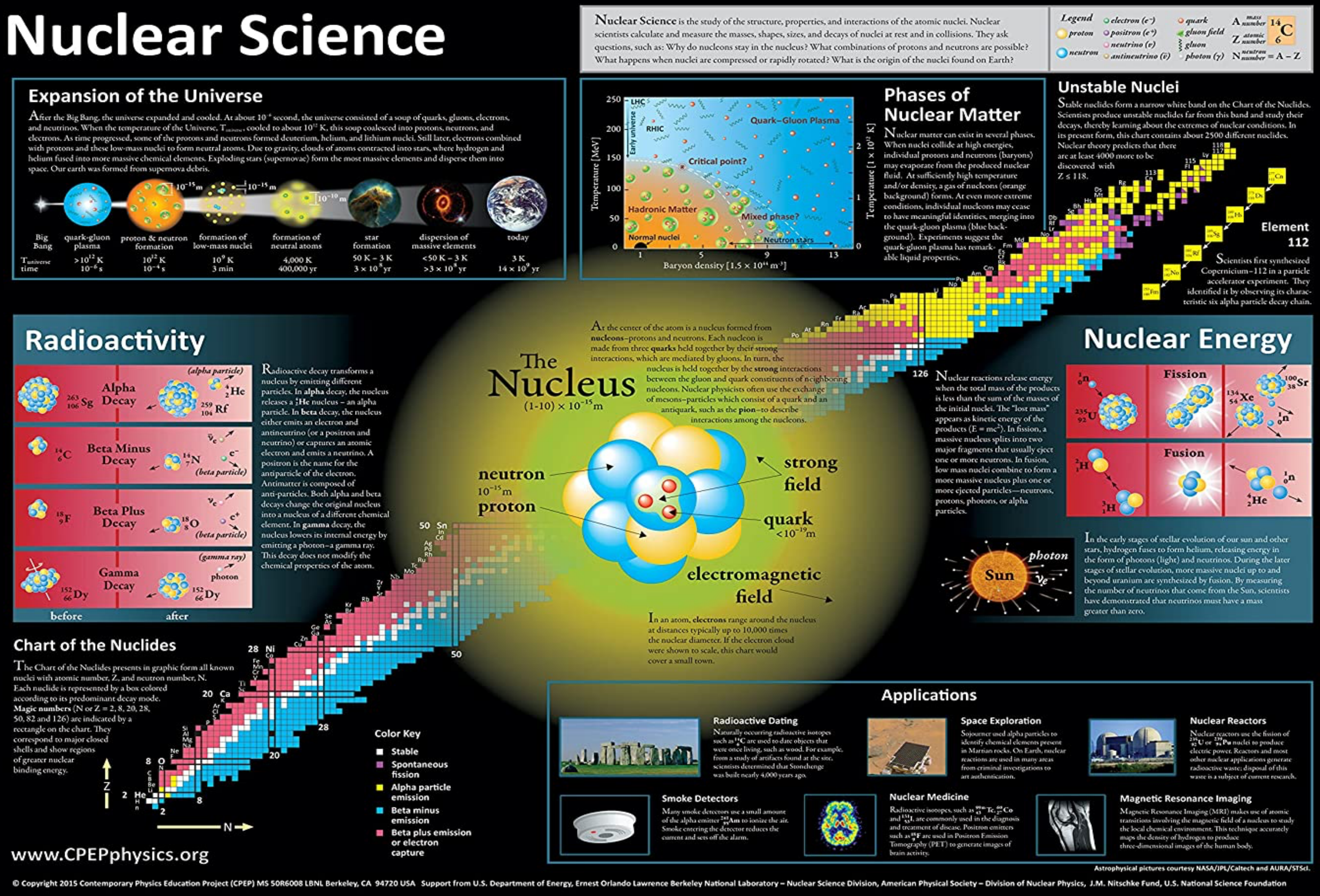
- 成绩

占比：平时考核（40%）、期中作业（30%）、期末作业（30%）

平时考核：作业、课堂**讨论**、小组学习汇报（分工明确）

关于核物理，我们知道什么？

关于核物理， 我们知道什么？



Nuclear Science

Nuclear Science is the study of the structure, properties, and interactions of the atomic nuclei. Nuclear scientists calculate and measure the masses, shapes, sizes, and decays of nuclei at rest and in collisions. They ask questions, such as: Why do nucleons stay in the nucleus? What combinations of protons and neutrons are possible? What happens when nuclei are compressed or rapidly rotated? What is the origin of the nuclei found on Earth?

Expansion of the Universe

After the Big Bang, the universe expanded and cooled. At about 10^{-6} second, the universe consisted of a soup of quarks, gluons, electrons, and neutrinos. When the temperature of the Universe, $T_{universe}$, cooled to about 10^{12} K, this soup coalesced into protons, neutrons, and electrons. As time progressed, some of the protons and neutrons formed deuterium, helium, and lithium nuclei. Still later, electrons combined with protons and these low-mass nuclei to form neutral atoms. Due to gravity, clouds of atoms contracted into stars, where hydrogen and helium fused into more massive chemical elements. Exploding stars (supernovae) form the most massive elements and disperse them into space. Our earth was formed from supernova debris.

Phases of Nuclear Matter

Nuclear matter can exist in several phases. When nuclei collide at high energies, individual protons and neutrons (baryons) may evaporate from the produced nuclear fluid. At sufficiently high temperature and/or density, a gas of nucleons (orange background) forms. At even more extreme conditions, individual nucleons may cease to have meaningful identities, merging into the quark-gluon plasma (blue background). Experiments suggest the quark-gluon plasma has remarkable liquid properties.

Unstable Nuclei

Stable nuclides form a narrow white band on the Chart of the Nuclides. Scientists produce unstable nuclides far from this band and study their decays, thereby learning about the extremes of nuclear conditions. In its present form, this chart contains about 2500 different nuclides. Nuclear theory predicts that there are at least 4000 more to be discovered with Z ≤ 118.

Element 112

Scientists first synthesized Copernicium–112 in a particle accelerator experiment. They identified it by observing its characteristic six alpha particle decay chain.

Radioactivity

Radioactive decay transforms a nucleus by emitting different particles. In **alpha** decay, the nucleus releases a $^{4}_{2}$**He** nucleus – an alpha particle. In **beta** decay, the nucleus either emits an electron and antineutrino (or a positron and neutrino) or captures an atomic electron and emits a neutrino. A positron is the name for the antiparticle of the electron. Antimatter is composed of anti-particles. Both alpha and beta decays change the original nucleus into a nucleus of a different chemical element. In **gamma** decay, the nucleus lowers its internal energy by emitting a photon–a gamma ray. This decay does not modify the chemical properties of the atom.

The Nucleus

$(1\text{-}10) \times 10^{-15}$m

At the center of the atom is a nucleus formed from **nucleons**–protons and neutrons. Each nucleon is made from three **quarks** held together by their strong interactions, which are mediated by gluons. In turn, the nucleus is held together by the **strong** interactions between the gluon and quark constituents of neighboring nucleons. Nuclear physicists often use the exchange of mesons–particles which consist of a quark and an antiquark, such as the **pion**–to describe interactions among the nucleons.

In an atom, **electrons** range around the nucleus at distances typically up to 10,000 times the nuclear diameter. If the electron cloud were shown to scale, this chart would cover a small town.

Nuclear Energy

Nuclear reactions release energy when the total mass of the products is less than the sum of the masses of the initial nuclei. The "lost mass" appears as kinetic energy of the products ($E = mc^2$). In fission, a massive nucleus splits into two major fragments that usually eject one or more neutrons. In fusion, low mass nuclei combine to form a more massive nucleus plus one or more ejected particles—neutrons, protons, photons, or alpha particles.

In the early stages of stellar evolution of our sun and other stars, hydrogen fuses to form helium, releasing energy in the form of photons (light) and neutrinos. During the later stages of stellar evolution, more massive nuclei up to and beyond uranium are synthesized by fusion. By measuring the number of neutrinos that come from the Sun, scientists have demonstrated that neutrinos must have a mass greater than zero.

Chart of the Nuclides

The Chart of the Nuclides presents in graphic form all known nuclei with atomic number, Z, and neutron number, N. Each nuclide is represented by a box colored according to its predominant decay mode. **Magic numbers** (N or Z = 2, 8, 20, 28, 50, 82 and 126) are indicated by a rectangle on the chart. They correspond to major closed shells and show regions of greater nuclear binding energy.

Color Key

- ■ Stable
- ■ Spontaneous fission
- ■ Alpha particle emission
- ■ Beta minus emission
- ■ Beta plus emission or electron capture

www.CPEPphysics.org

Applications

Radioactive Dating

Naturally occurring radioactive isotopes such as $^{14}_{6}C$ are used to date objects that were once living, such as wood. For example, from a study of artifacts found at the site, scientists determined that Stonehenge was built nearly 4,000 years ago.

Space Exploration

Sojourner used alpha particles to identify chemical elements present in Martian rocks. On Earth, nuclear reactions are used in many areas from criminal investigations to art authentication.

Nuclear Reactors

Nuclear reactors use the fission of $^{235}_{92}U$ or $^{239}_{94}Pu$ nuclei to produce electric power. Reactors and most other nuclear applications generate radioactive waste; disposal of this waste is a subject of current research.

Smoke Detectors

Many smoke detectors use a small amount of the alpha emitter $^{241}_{95}Am$ to ionize the air. Smoke entering the detector reduces the current and sets off the alarm.

Nuclear Medicine

Radioactive isotopes, such as $^{99m}_{43}Tc$, $^{60}_{27}Co$ and $^{131}_{53}I$, are commonly used in the diagnosis and treatment of disease. Positron emitters such as $^{18}_{9}F$ are used in Positron Emission Tomography (PET) to generate images of brain activity.

Magnetic Resonance Imaging

Magnetic Resonance Imaging (MRI) makes use of atomic transitions involving the magnetic field of a nucleus to study the local chemical environment. This technique accurately maps the density of hydrogen to produce three-dimensional images of the human body.

Astrophysical pictures courtesy NASA/JPL/Caltech and AURA/STScI.

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关于核物理，我们知道什么？

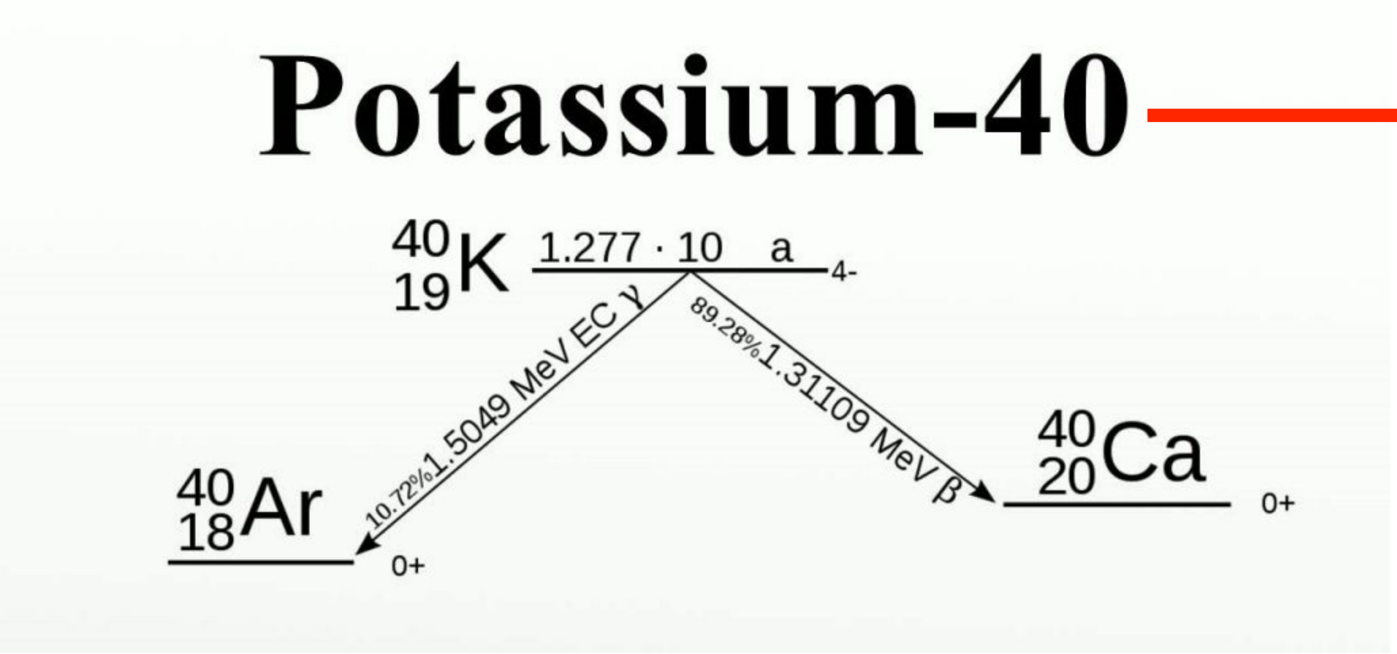
- 原子核基本组分？如何得知？
- 原子核多大？如何测定？
- 原子核哪来的？太阳与核电站都提供能源，有什么共同点和不同点？
- 自然界稳定原子核有多少？为什么只有这些？为什么“金”贵”铁“便宜？
- “点石成金”可能实现吗？
- 原子核中核子数目有上限吗？为什么？
- 核子有内部结构吗？如何知道？
- 自然界都有哪些基本粒子，参与哪些基本相互作用？
- 恒星燃烧后变成什么？他们与核物理有什么关系？
- 引力波探测项目与核物理学研究有关系吗？
- 。。。

身边的核物理 (举例)

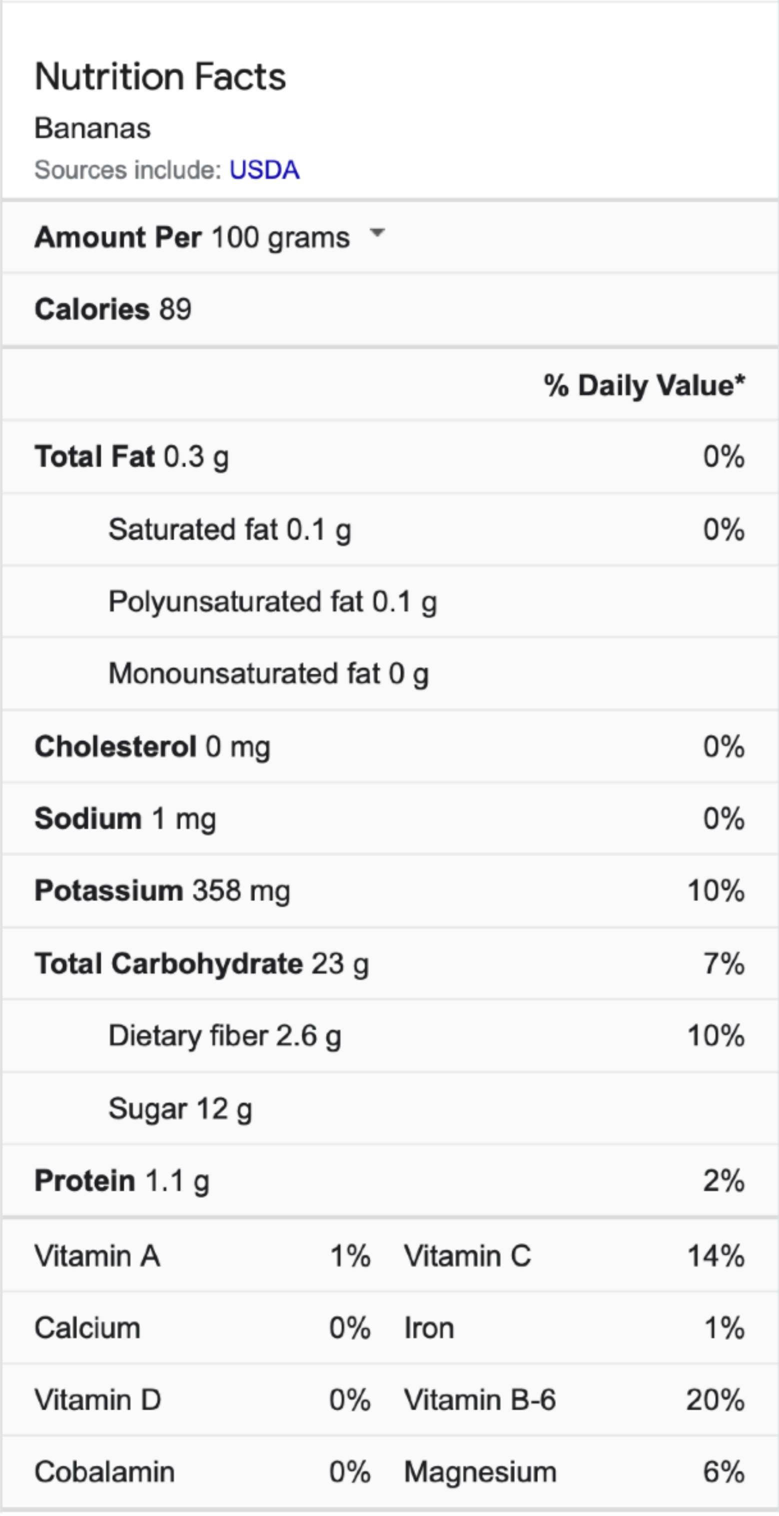
Banana Equivalent Dose

A unit that compares radiation exposure to the amount you get naturally from eating a banana.

1 banana = 0.1 μSv



Potassium-40



Nutrition Facts
Bananas
Sources include: USDA

Amount Per 100 grams

Calories 89

	% Daily Value*
Total Fat 0.3 g	0%
Saturated fat 0.1 g	0%
Polyunsaturated fat 0.1 g	
Monounsaturated fat 0 g	
Cholesterol 0 mg	0%
Sodium 1 mg	0%
Potassium 358 mg	10%
Total Carbohydrate 23 g	7%
Dietary fiber 2.6 g	10%
Sugar 12 g	
Protein 1.1 g	2%

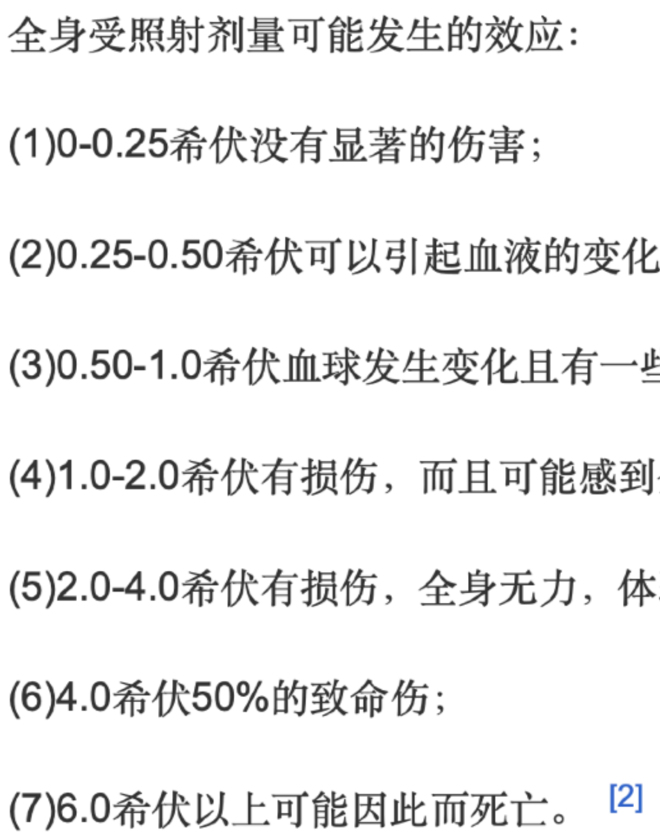
Vitamin A	1%	Vitamin C	14%
Calcium	0%	Iron	1%
Vitamin D	0%	Vitamin B-6	20%
Cobalamin	0%	Magnesium	6%

*Per cent Daily Values are based on a 2,000 calorie diet.

从上面这张图中您了解到什么信息?

Calcium Ca2+ is found in food in different heavy **isotopes**, e.g. 42Ca or 44Ca.

身边的核物理 (举例)



全身受照射剂量可能发生的效应：

(1)0-0.25希伏没有显著的伤害；

(2)0.25-0.50希伏可以引起血液的变化，但无严重伤害；

(3)0.50-1.0希伏血球发生变化且有一些损害，但无疲劳感；

(4)1.0-2.0希伏有损伤，而且可能感到全身无力；

(5)2.0-4.0希伏有损伤，全身无力，体弱者可能死亡；

(6)4.0希伏50%的致命伤；

(7)6.0希伏以上可能因此而死亡。 [2]

輻射劑量（mSv）	影響和標準
0.001	手足X射線照相
0.001	骨密度X射線检查
0.005–0.01	口腔X射線照相
0.01	四肢X射線照相
0.02–0.1	做一次X射線胸部攝片的劑量。另说为0.9mSv[7]
0.2	乘飛機從東京到紐約之間往返一次的劑量（宇宙射線和飛行高度有關）。
0.4	筛查乳腺癌的钼靶检查（MAMMOGRAM）
0.54	腹部X射線照相
0.66	骨盆X射線照相
1.0	一般公衆一年工作所受人工放射劑量（ICRP推薦） 從事輻射相關工作的婦女從被告知懷孕到臨產所受人工放射劑量上限。
1.2	與1天平均吸1.5盒（30支）紙煙同居的被動吸煙者一年纍計輻射[8]。
1.5	日本人一年纍計所受自然輻射。
1.5	腰椎X射線照相
1.5	胸部低剂量CT筛查
2.0	從事輻射相關工作的婦女從被告知懷孕到臨產腹部表面所受人工放射劑量極限。
2.0	1次头部CT檢查
2.4	地球人平均一年纍計所受輻射（宇宙射線0.4，大地0.5，氡1.2，食物0.3）
2.5–4	一次胃部X射線鋇餐透視的劑量。
5	從事輻射相關的婦女工作者一年纍計所受輻射法定極限。
6	上消化道X射線檢查
6.9	1次胸部CT檢查
7.1	做一次X射線胸部透視的劑量。[7]
8	下消化道X射線檢查
10	日本原子力安全委員會所制定「室内避難」的輻射劑量。
10	一次腹腔骨盆CT检查
10 – 20	全身CT检查
13 – 60	1天平均吸1.5盒（30支）紙煙者一年纍計[8][9]。
20	放射性职业工作者一年累积全身受职业照射的上限（ICRP推荐）
50	從事輻射相關工作者（非女性）一年纍計所受輻射舊標准規定的上限。 日本原子力安全委員會所制定「避難」的輻射劑量。 自衛隊員，消防員，警察（婦女除外）一年纍計所受輻射法定極限。
100	已證明對人體健康明顯有害的輻射劑量極限 從事輻射相關工作者（非女性）五年纍計所受輻射法定極限。 從事輻射相關工作者（非女性）在緊急狀況下從事一次作業所受輻射法定極限[10]。
250	福島第一核電站事故現場人員暫定輻射劑量上限。 白血球減少。
500	淋巴球減少。 國際放射防護委員会規定除人命救援外所能承受的輻射極限。
500	放射性职业工作者一年累积局部（如皮肤、手、足）受职业照射的上限（ICRP推荐）
1,000	出現被輻射症狀。噁心，嘔吐。水晶體渾濁。
2,000	細胞組織遭破壞，内部出血，脫毛脫髮。死亡率5%。
3,000 – 5,000	死亡率50%（局部被輻射時3,000 :脫毛脫髮、4,000 :失去生育能力、5,000 : 白内障、皮膚出現紅斑）[11]。
7,000 – 10,000	死亡率99%。
10,001以上	死亡率99.9%
50,000	死亡率100%

10 个香蕉

1000个香蕉

10次胸透

100次胸透

15次全身CT

100次全身CT

数据源自维基百科

主要内容

✦ 原子核的结合能

- 原子核基本组分
- 原子核的结合能
- 液滴模型

✦ 原子核的状态量子数

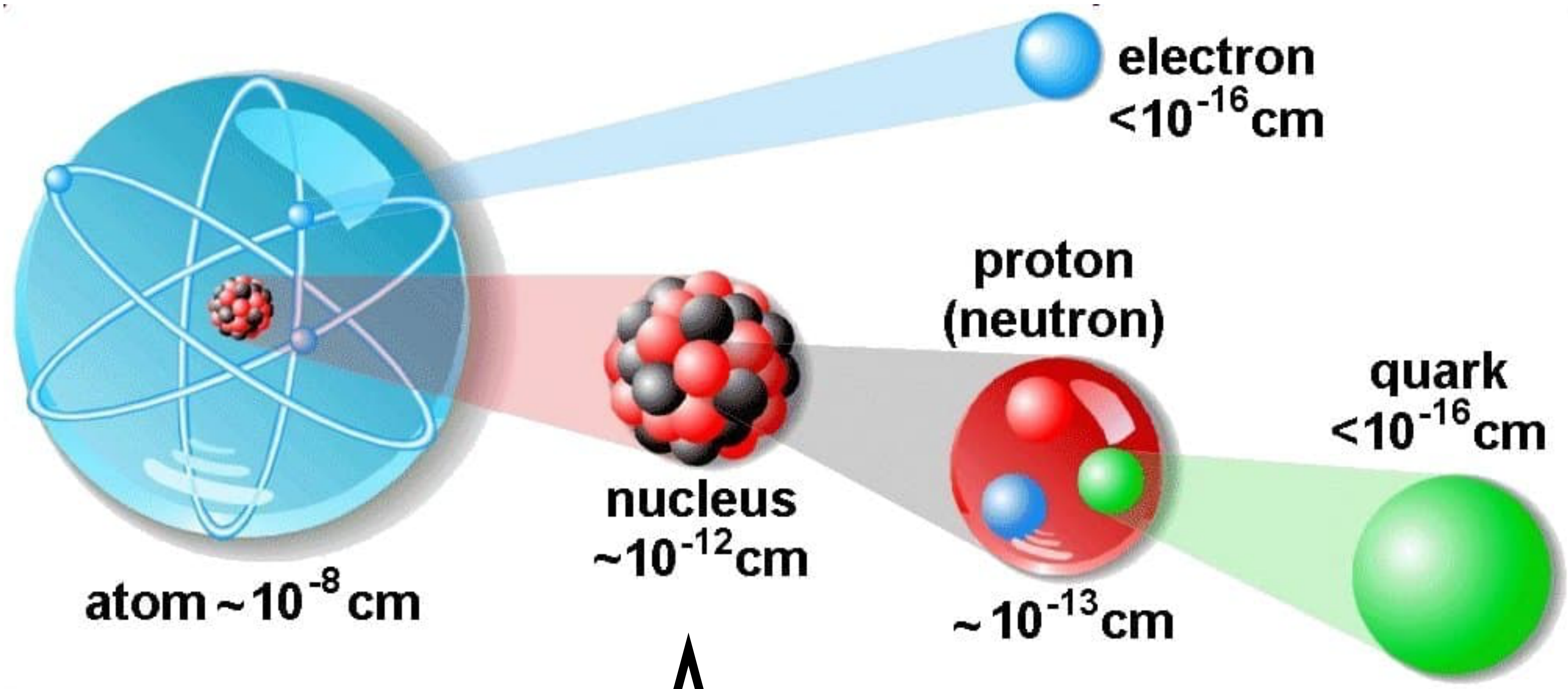
- 原子核的自旋宇称
- 原子核的同位旋

✦ 原子核的电磁矩

- 原子核的电多极矩
- 原子核的磁多极矩

原子核的基本组分

- 原子核由核子（质子、中子）组成

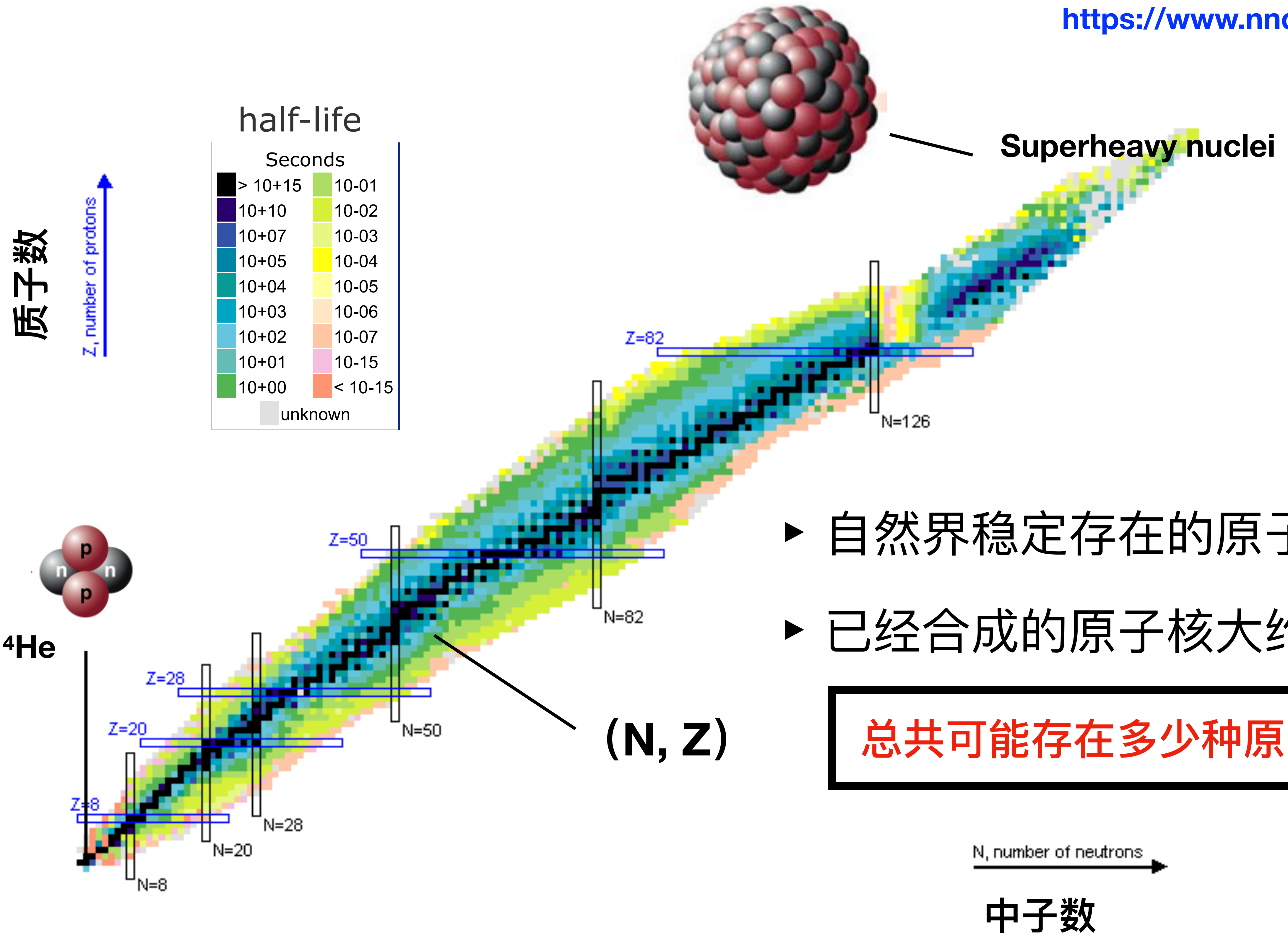


🔴 **Proton**

⚫ **Neutron**

原子核的基本组分

https://www.nndc.bnl.gov/nudat2/

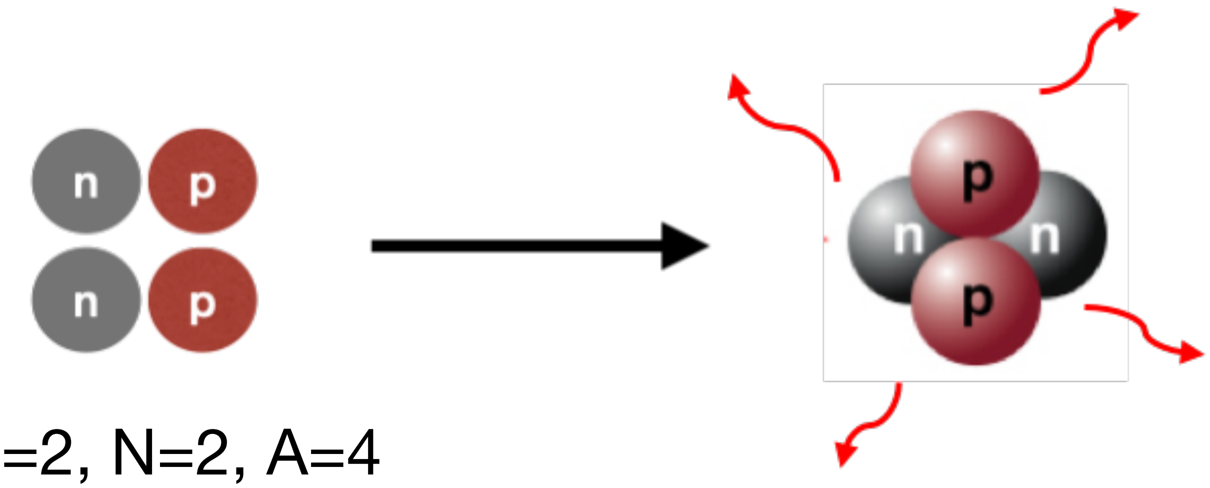


- 自然界稳定存在的原子核<300个
- 已经合成的原子核大约3300+个

总共可能存在多少种原子核?

原子核的结合能

- 原子核结合能：核子由自由状态结合成束缚的原子核放出的能量



Z=2, N=2, A=4

Z: 质子的数目
N: 中子的数目
A: 质量数，核子总数（=Z+N）

中子质量

$$B(Z,A) = ZM_H + (A-Z)M_n - M(Z,A)$$

相应原子质量

$$M_H \approx M_p + m_e$$

Electron binding energy

D. Lunney et al., *Rev. Mod. Phys.* **75,** 1021 (2003)

$$B_e(Z) = 14.4381Z^{2.39} + 1.55468\times 10^{-6}Z^{5.35}\ \mathrm{eV}.$$

$$\begin{aligned} M_\mathrm{p} &= 938.272\ \mathrm{MeV}/c^2 = 1{,}836.153\ m_\mathrm{e} \\ M_\mathrm{n} &= 939.565\ \mathrm{MeV}/c^2 = 1{,}838.684\ m_\mathrm{e} \\ m_\mathrm{e} &= \quad 0.511\ \mathrm{MeV}/c^2. \end{aligned}$$

$$\begin{aligned} \mathcal{M}(Z,A) &= M(Z,A) - [Zm_e + B_e(Z)] \\ &\approx M(Z,A) - Zm_e \end{aligned}$$

原子核质量

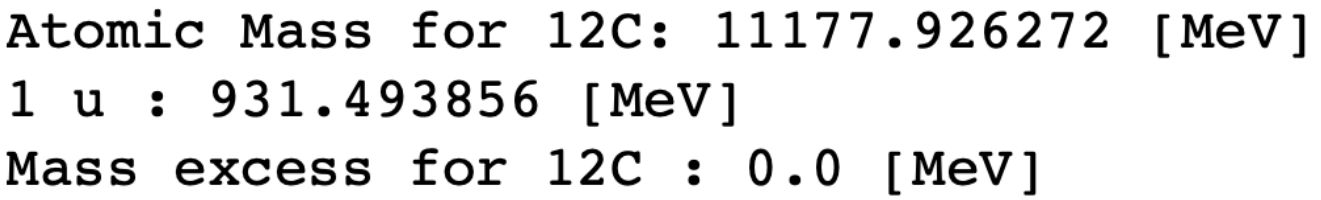
原子核的结合能

- 原子（核）质量（Nuclear Mass）：

$$M(Z,A) = ZM_H + (A-Z)M_n - B(Z,A)$$

- 原子（核）质量过剩（Mass excess）：

$$\Delta M(Z,A) = M(A,Z) - Am_u$$



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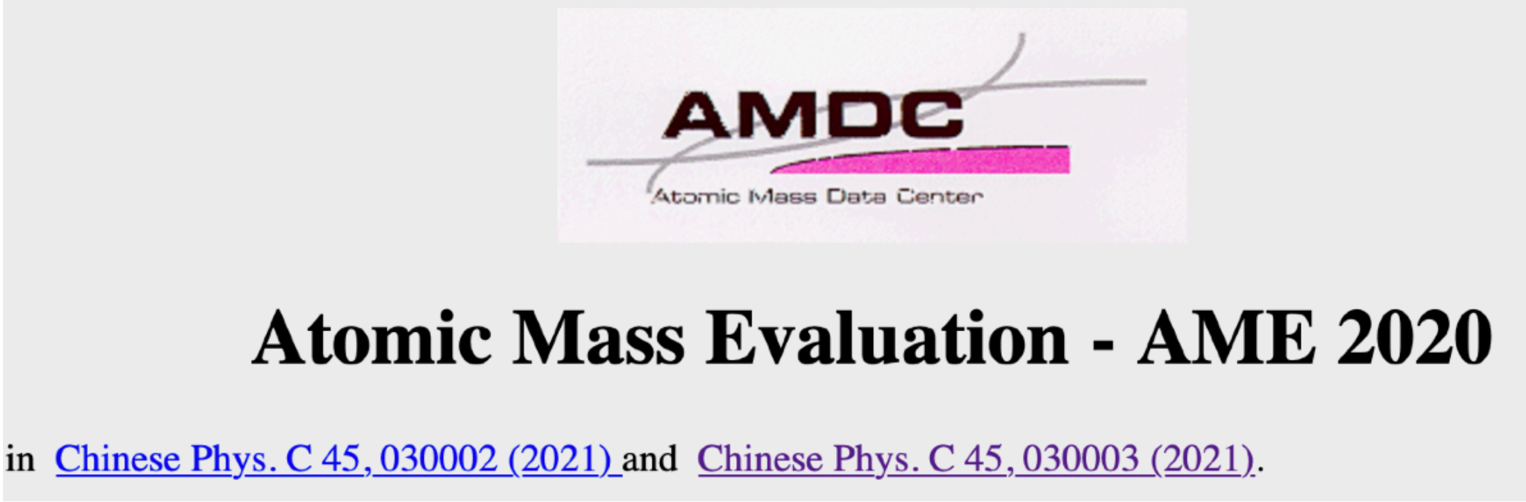
Atomic Mass for 12C: 11177.926272 [MeV]
1 u : 931.493856 [MeV]
Mass excess for 12C : 0.0 [MeV]
```

原子质量单位 (u)： ^{12}C原子质量/12

For ^{12}C, $\Delta M(Z,A) = 0$

http://amdc.impcas.ac.cn/web/masseval.html



Atomic Mass Evaluation - AME 2020

in Chinese Phys. C 45, 030002 (2021) and Chinese Phys. C 45, 030003 (2021).

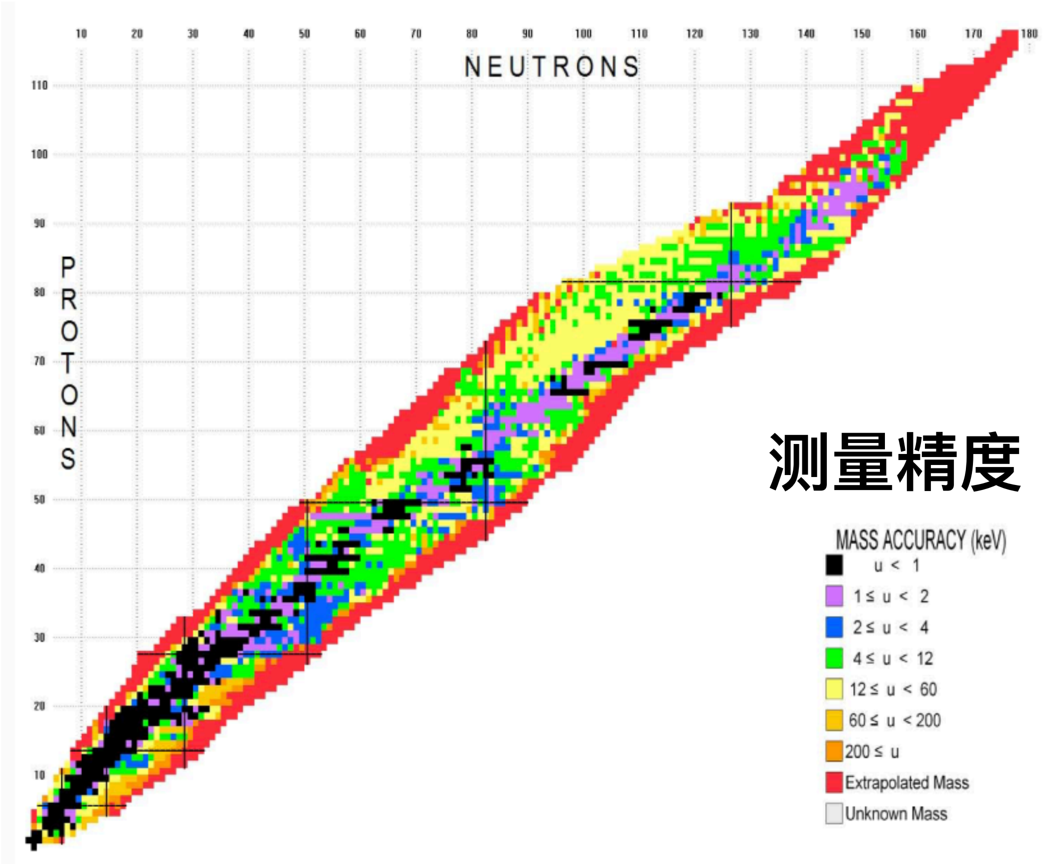
原子核的结合能

- 原子核结合能实验数据

https://www.nndc.bnl.gov

https://waww-nds.iaea.org/amdc/

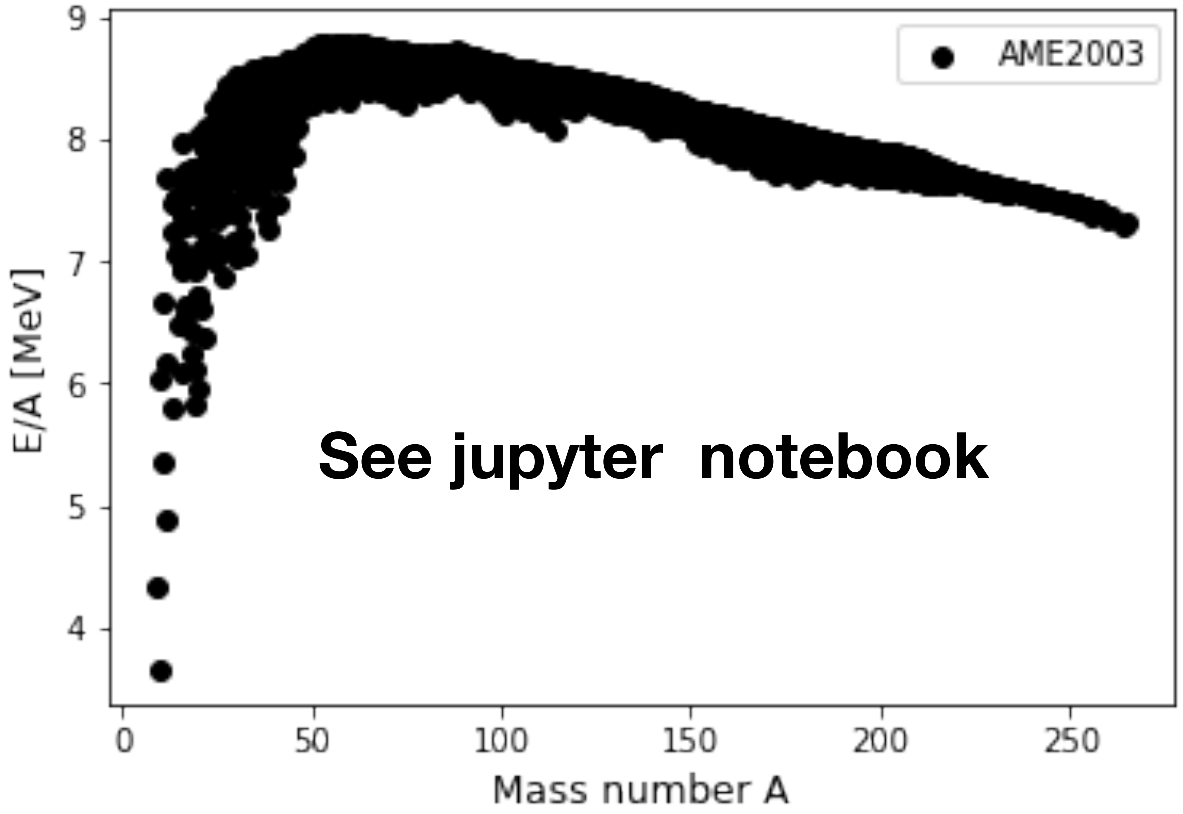
http://www.nuclear.csdb.cn



测量精度

Figure 1. Chart of the nuclides displaying the accuracy 'u' of masses (created by NUCLEUS-AMDC).

G. Audi et al., Chinese Physics C Vol. 41, No. 3 (2017) 030001



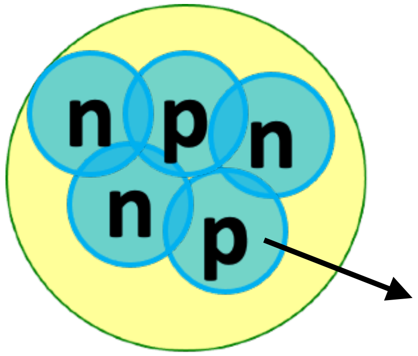
See jupyter notebook

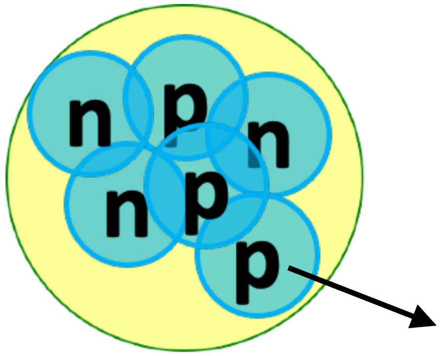
原子核的结合能

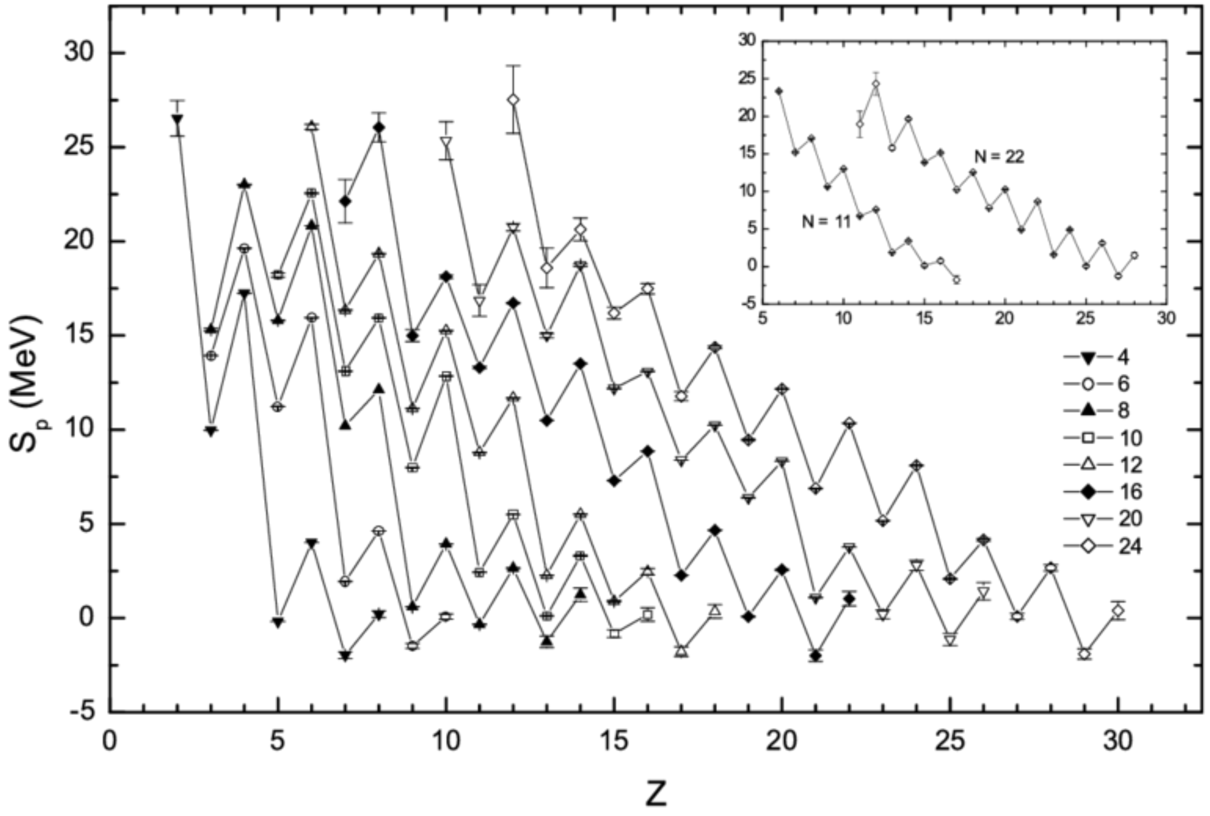
- 质子分离能

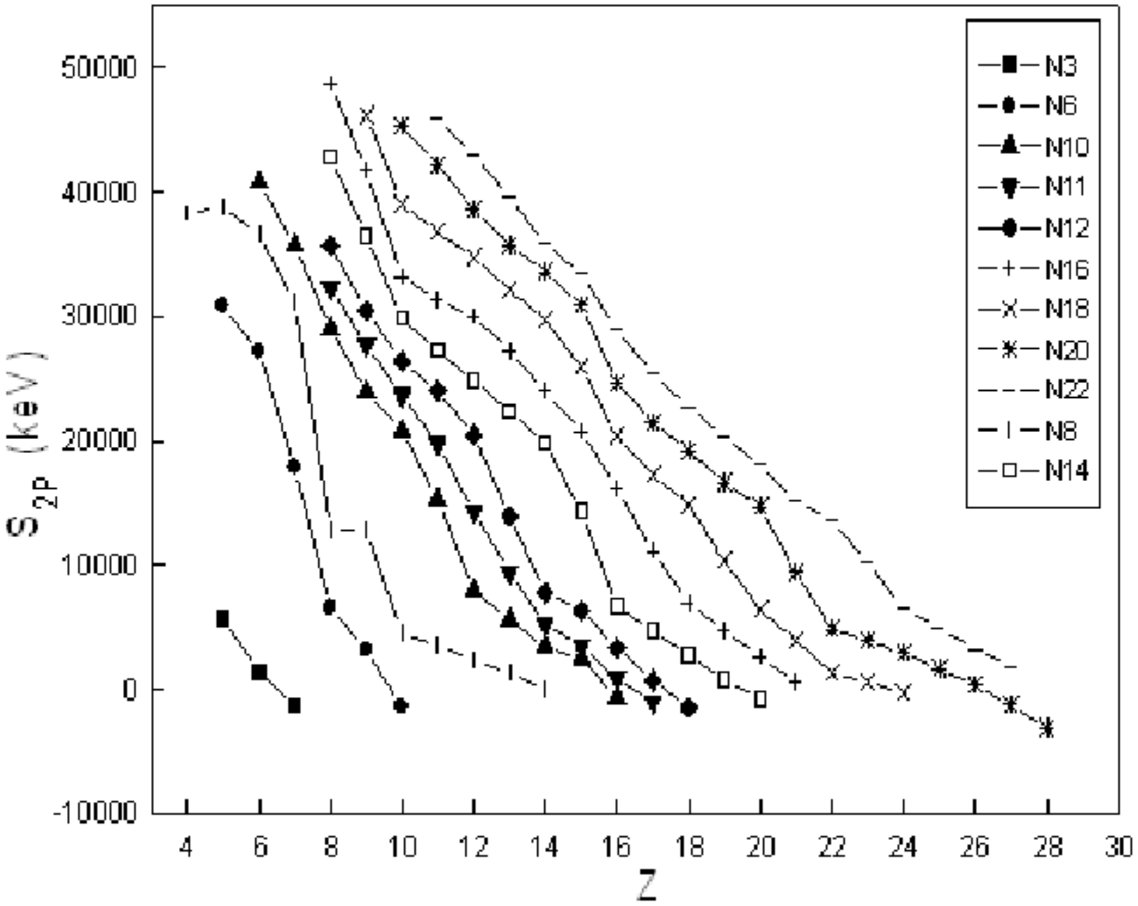
单质子分离能 S_p： $$S_p = B(A,Z) - B(A-1,Z-1)$$

双质子分离能 S_{2p}： $$S_{2p} = B(A,Z) - B(A-2,Z-2)$$









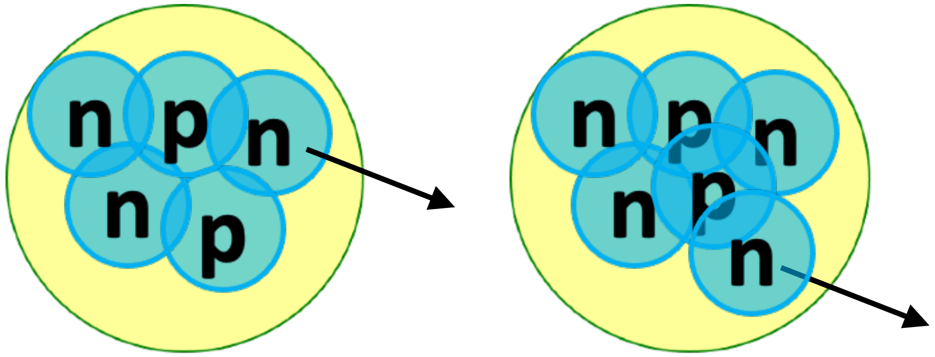
- Z为偶数原子核的单质子 > Z为奇数原子核的单质子 （配对效应）。
- 双质子分离能在原子核具有某些Z数值时下降迅速 （壳结构效应）。

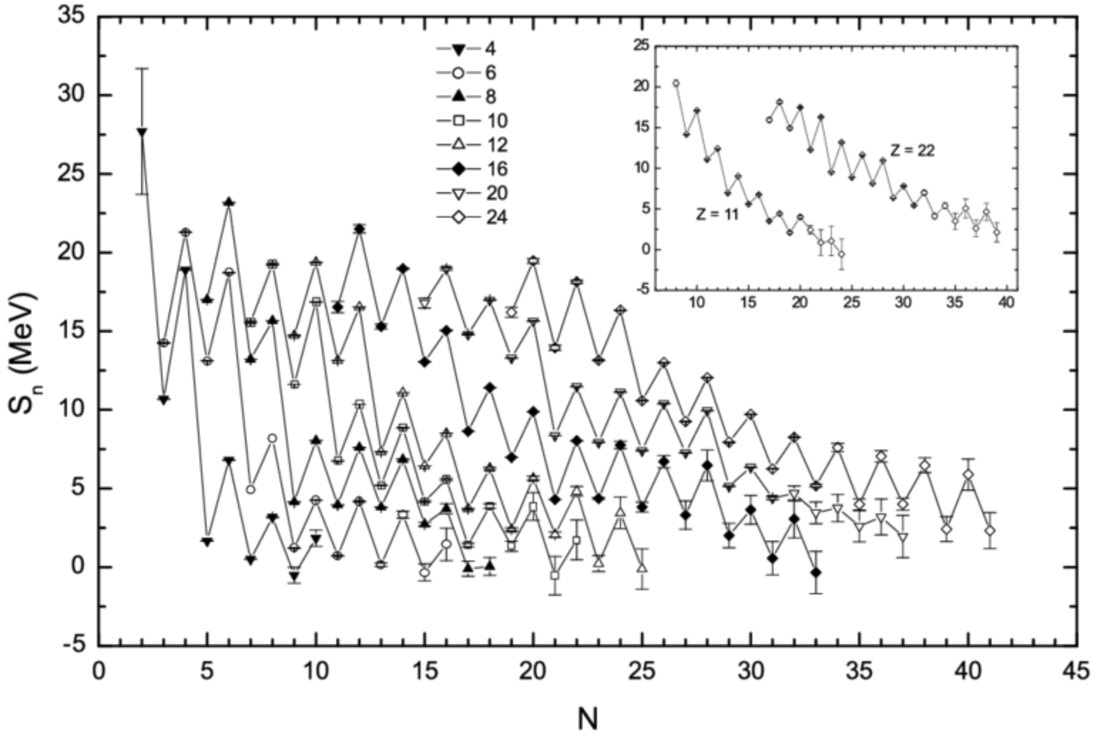
原子核的结合能

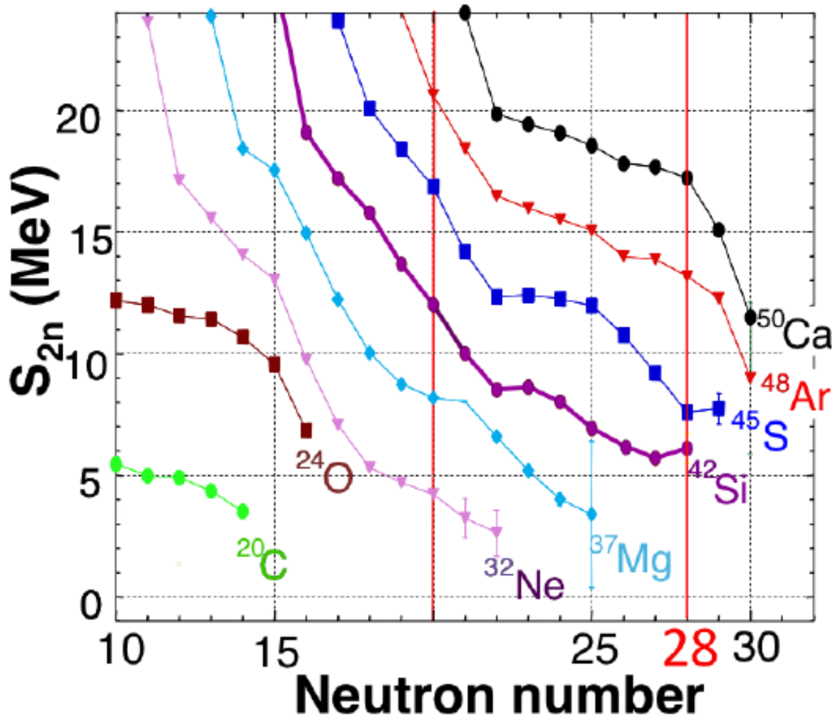
- 中子分离能

单中子分离能 S_n： $S_n = B(A,Z) - B(A-1,Z)$

双中子分离能 S_{2n}： $S_{2n} = B(A,Z) - B(A-2,Z)$



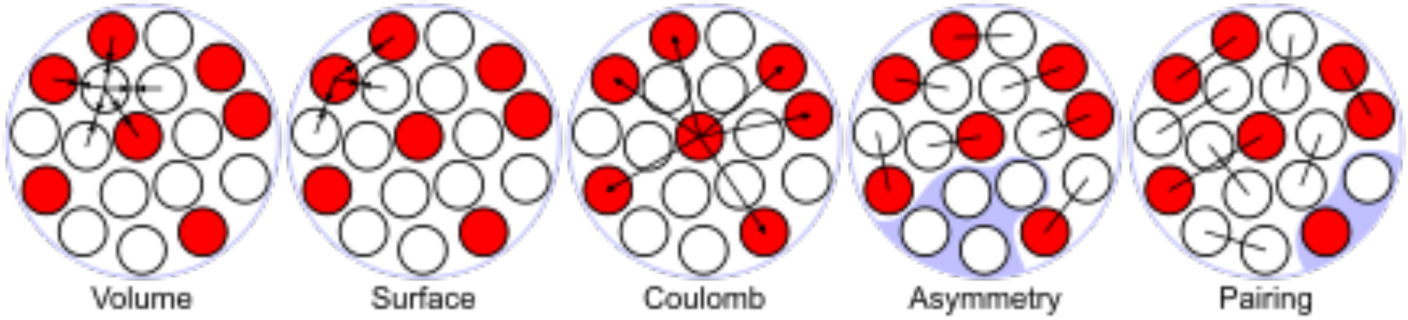




- N为偶数原子核的单中子 > N为奇数原子核的单中子 （配对效应）。
- 双中子分离能在原子核具有某些N数值时下降迅速 （壳结构效应）。

原子核的液滴模型

- Weizsäcker 公式 （半经验质量公式）：

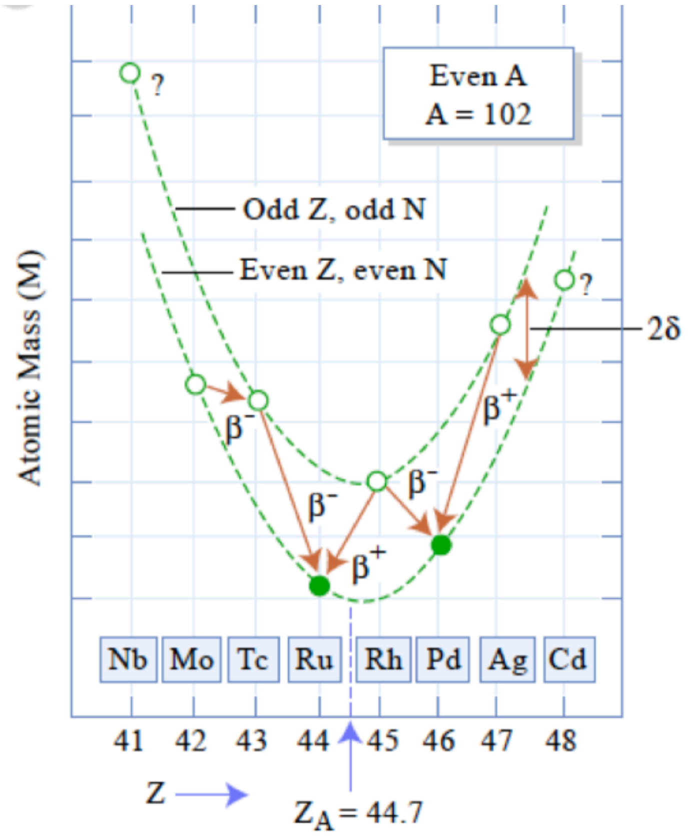


The Semi-Empirical Mass Formula

$$B(A,Z) = a_V \cdot A - a_S \cdot A^{2/3} - a_C \cdot \frac{Z \cdot (Z-1)}{A^{1/3}} - a_{asym} \cdot \frac{(A-2Z)^2}{A} + a_{pair} \cdot \frac{\delta}{A^{1/2}}$$

参数化：

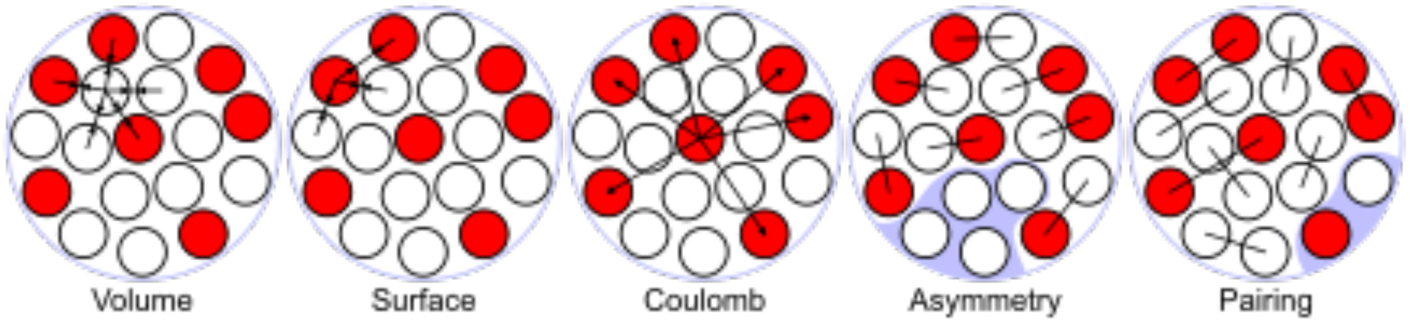
a_V	15.85 MeV
a_S	18.34 MeV
a_C	0.71 MeV
a_{asym}	23.21 MeV
a_{pair}	12 MeV



$$\delta = \begin{cases} +1 & for\ even-even\ nuclei \\ 0 & for\ odd-even\ nuclei \\ -1 & for\ odd-odd\ nuclei \end{cases}$$

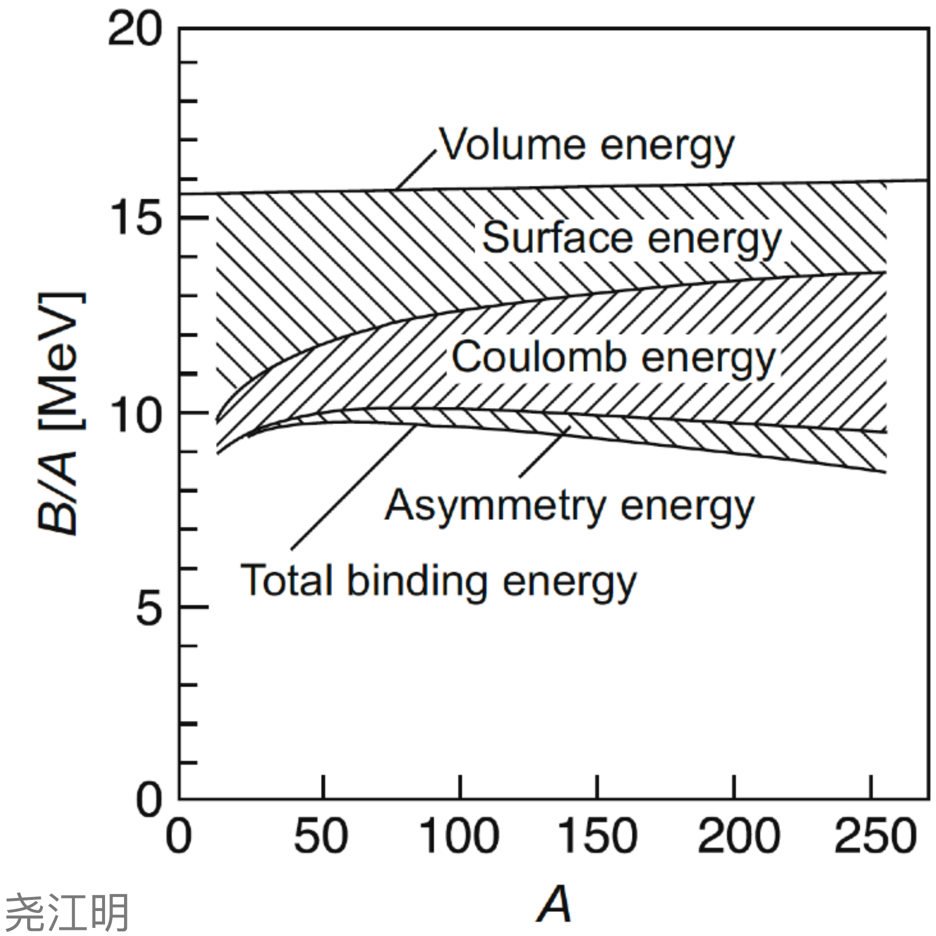
原子核的液滴模型

- Weizsäcker 公式 （半经验质量公式）：



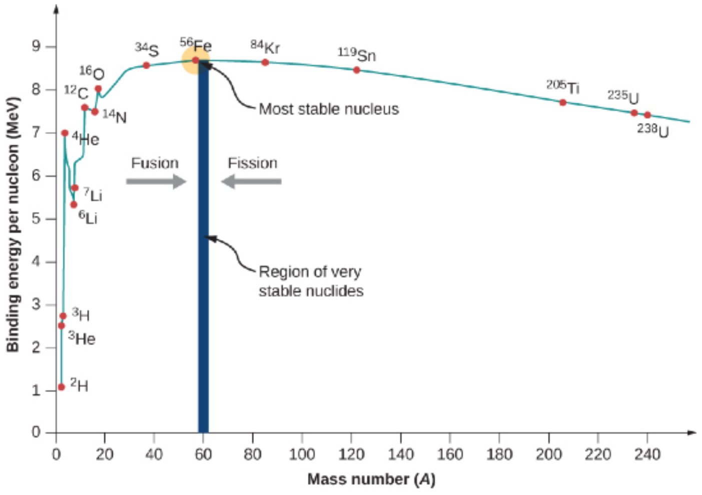
The Semi-Empirical Mass Formula

$$B(A,Z) = a_V \cdot A - a_S \cdot A^{2/3} - a_C \cdot \frac{Z \cdot (Z-1)}{A^{1/3}} - a_{asym} \cdot \frac{(A-2Z)^2}{A} + a_{pair} \cdot \frac{\delta}{A^{1/2}}$$



体积能
表面能
库伦能
对称能
总能量

$$\delta = \begin{cases} +1 & for\ even-even\ nuclei \\ 0 & for\ odd-even\ nuclei \\ -1 & for\ odd-odd\ nuclei \end{cases}$$



原子核的液滴模型

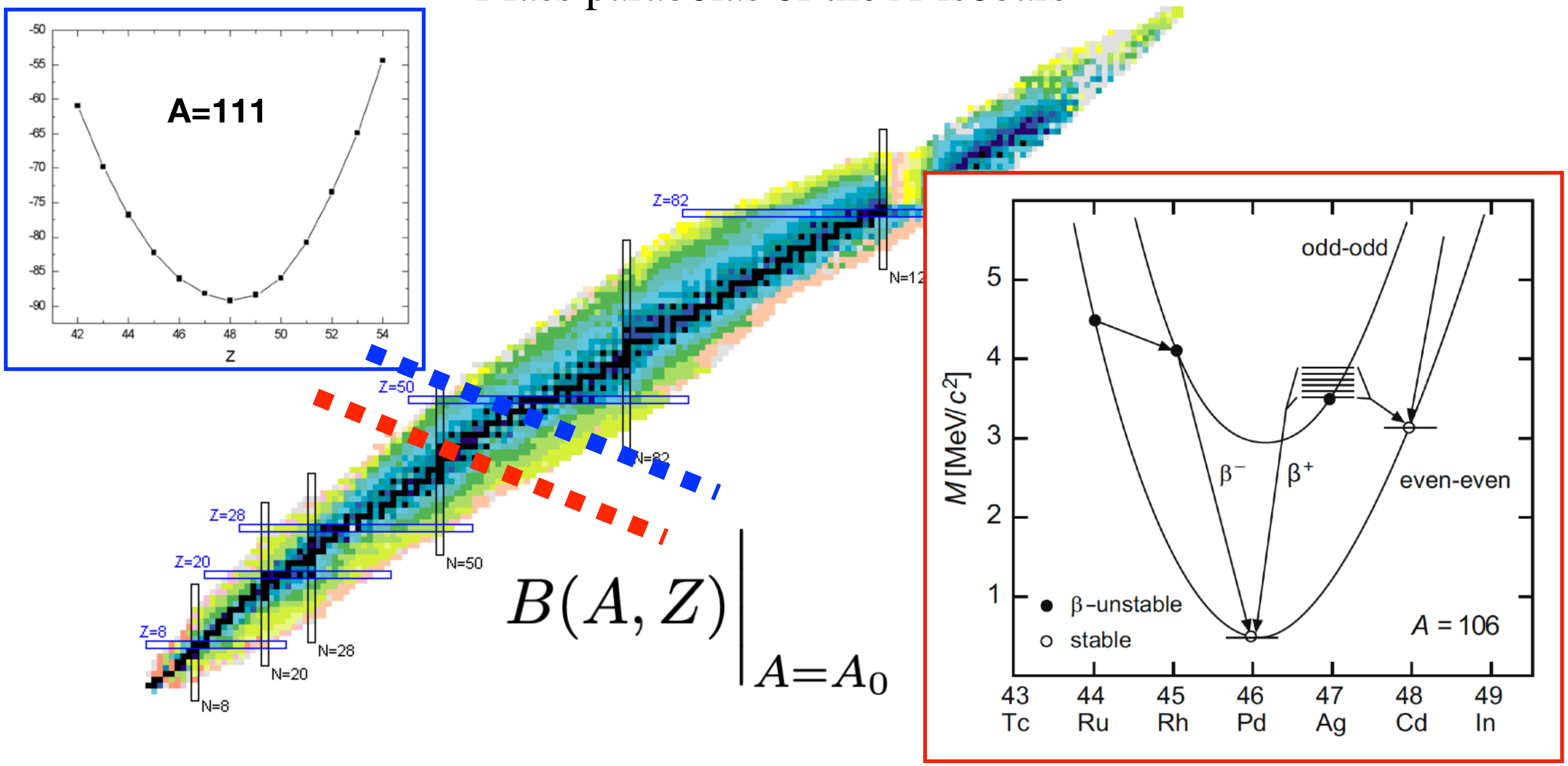
- Weizsäcker 公式 （半经验质量公式）：

The Semi-Empirical Mass Formula

$$B(A,Z) = a_V \cdot A - a_S \cdot A^{2/3} - a_C \cdot \frac{Z \cdot (Z-1)}{A^{1/3}} - a_{asym} \cdot \frac{(A-2Z)^2}{A} + a_{pair} \cdot \frac{\delta}{A^{1/2}}$$

Odd-even nuclei

Mass parabolas of the A-isobars



$$B(A,Z)\Big|_{A=A_0}$$

原子核的液滴模型

- Weizsäcker 公式 （半经验质量公式）：

The Semi-Empirical Mass Formula

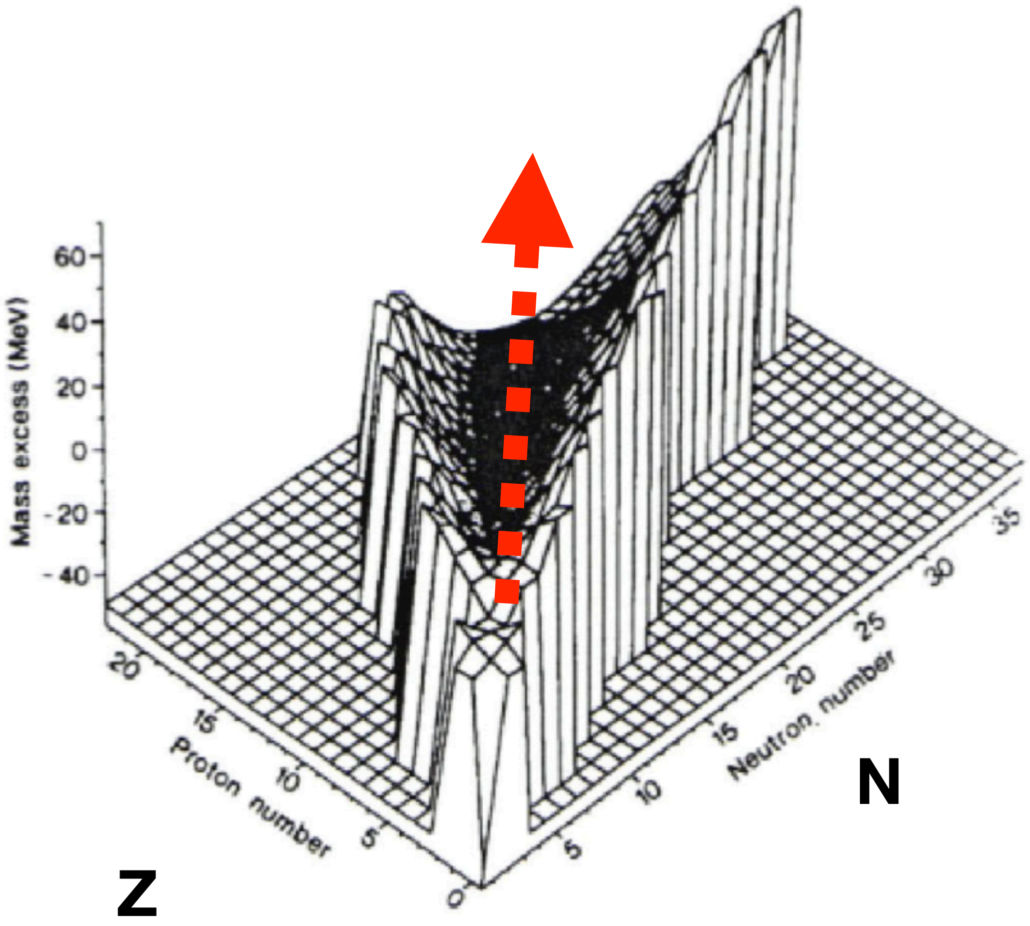
$$B(A,Z) = a_V \cdot A - a_S \cdot A^{2/3} - a_C \cdot \frac{Z \cdot (Z-1)}{A^{1/3}} - a_{asym} \cdot \frac{(A-2Z)^2}{A} + a_{pair} \cdot \frac{\delta}{A^{1/2}}$$

$$\left.\frac{\partial B(A,Z)}{\partial Z}\right|_{A=A_0} = 0$$

Beta-stability line

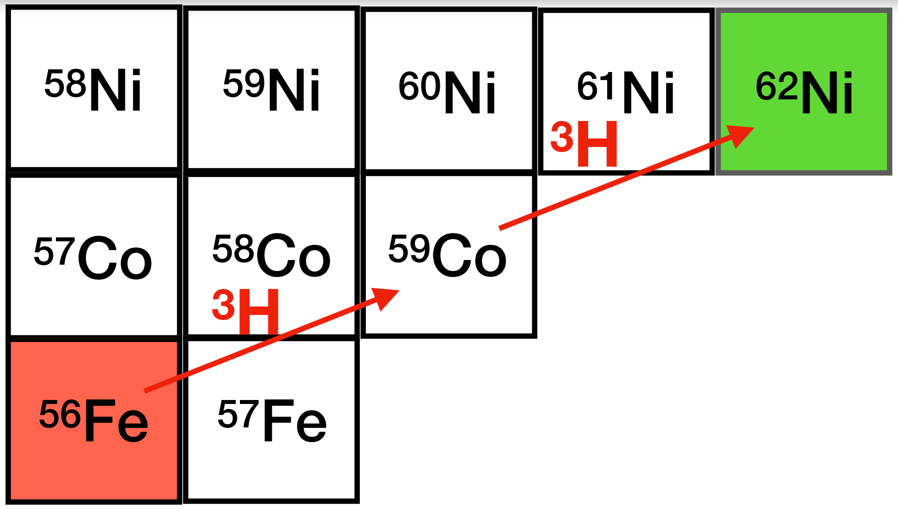
$$Z = \frac{A_0}{1.98 + 0.0155 A_0^{2/3}}$$

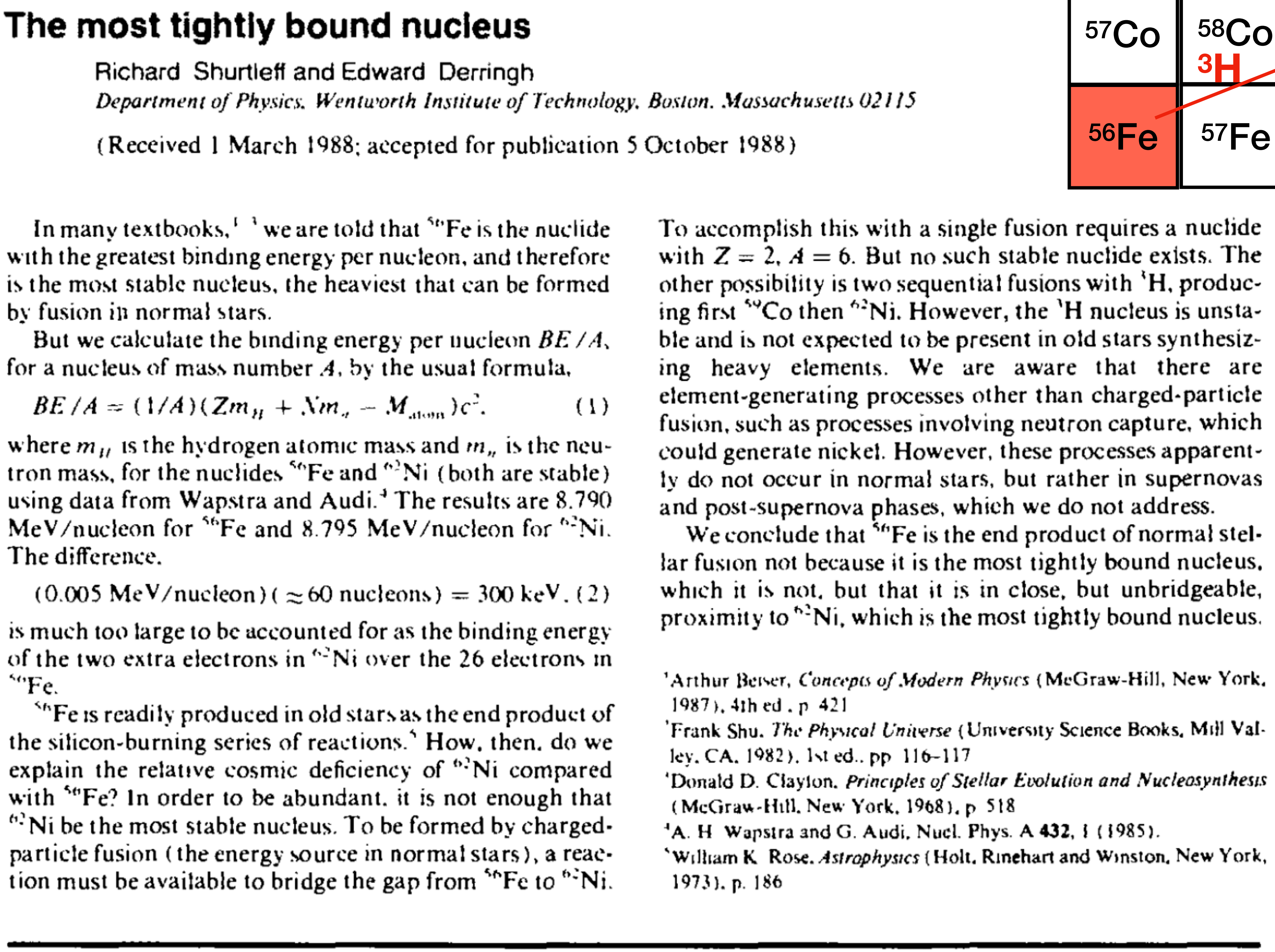
Depends on parameterization (Homework)



原子核的液滴模型

- ^{62}Ni is the most tightly bound nucleus
- ^{56}Fe is more abundant than ^{62}Ni





The most tightly bound nucleus

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(Received 1 March 1988; accepted for publication 5 October 1988)

In many textbooks,[1-3] we are told that ^{56}Fe is the nuclide with the greatest binding energy per nucleon, and therefore is the most stable nucleus, the heaviest that can be formed by fusion in normal stars.

But we calculate the binding energy per nucleon BE/A, for a nucleus of mass number A, by the usual formula,

$$BE/A = (1/A)(Zm_H + Nm_n - M_{\text{atom}})c^2. \qquad (1)$$

where m_H is the hydrogen atomic mass and m_n is the neutron mass, for the nuclides ^{56}Fe and ^{62}Ni (both are stable) using data from Wapstra and Audi.[4] The results are 8.790 MeV/nucleon for ^{56}Fe and 8.795 MeV/nucleon for ^{62}Ni. The difference.

$$(0.005\ \text{MeV/nucleon})(\approx 60\ \text{nucleons}) = 300\ \text{keV}. \qquad (2)$$

is much too large to be accounted for as the binding energy of the two extra electrons in ^{62}Ni over the 26 electrons in ^{56}Fe.

^{56}Fe is readily produced in old stars as the end product of the silicon-burning series of reactions.[5] How, then, do we explain the relative cosmic deficiency of ^{62}Ni compared with ^{56}Fe? In order to be abundant, it is not enough that ^{62}Ni be the most stable nucleus. To be formed by charged-particle fusion (the energy source in normal stars), a reaction must be available to bridge the gap from ^{56}Fe to ^{62}Ni. To accomplish this with a single fusion requires a nuclide with $Z = 2$, $A = 6$. But no such stable nuclide exists. The other possibility is two sequential fusions with ^{3}H, producing first ^{59}Co then ^{62}Ni. However, the ^{3}H nucleus is unstable and is not expected to be present in old stars synthesizing heavy elements. We are aware that there are element-generating processes other than charged-particle fusion, such as processes involving neutron capture, which could generate nickel. However, these processes apparently do not occur in normal stars, but rather in supernovas and post-supernova phases, which we do not address.

We conclude that ^{56}Fe is the end product of normal stellar fusion not because it is the most tightly bound nucleus, which it is not, but that it is in close, but unbridgeable, proximity to ^{62}Ni, which is the most tightly bound nucleus.

---

[1] Arthur Beiser, *Concepts of Modern Physics* (McGraw-Hill, New York, 1987), 4th ed., p. 421.
[2] Frank Shu, *The Physical Universe* (University Science Books, Mill Valley, CA, 1982), 1st ed., pp. 116–117.
[3] Donald D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis* (McGraw-Hill, New York, 1968), p. 518.
[4] A. H. Wapstra and G. Audi, Nucl. Phys. A **432**, 1 (1985).
[5] William K. Rose, *Astrophysics* (Holt, Rinehart and Winston, New York, 1973), p. 186.

中场休息

核物理概论
第一章：原子核整体性质

原子核的状态量子数

- 原子核的自旋、宇称 J^π

$$\hat{J} = \sum_{i=1}^{A} \hat{j}_i,$$

$$\pi = (-1)^{\sum_i l_i}$$

单个核子自旋矢量叠加 $\hat{j} = \hat{\ell} + \hat{s},$

基态自旋宇称：

偶偶核：0+
奇偶核：半整数自旋
奇奇核：整数自旋

$$\hat{J} = \hat{j}_{\mathrm{p}} + \hat{j}_{\mathrm{n}}.$$

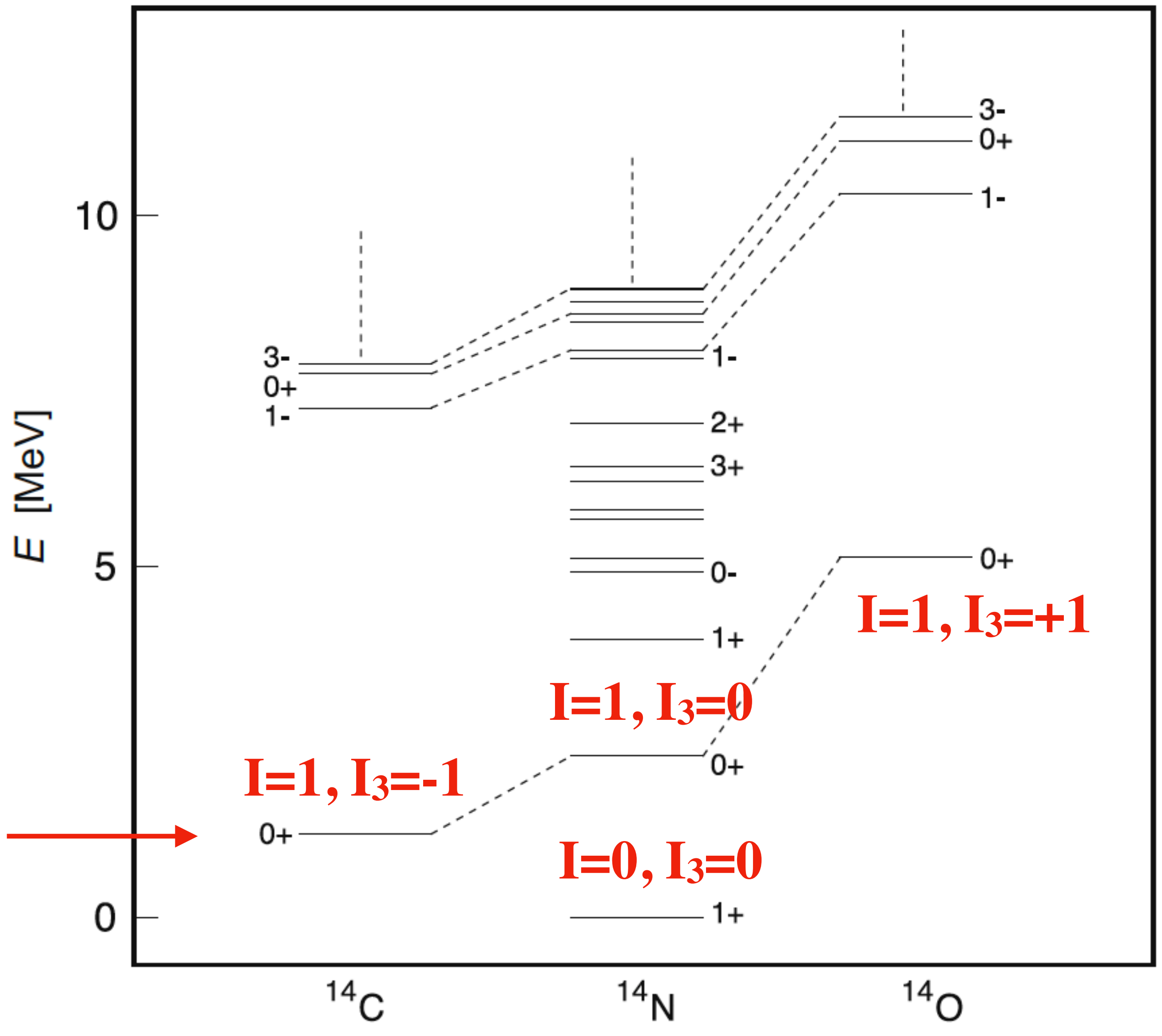
原子核的状态量子数

- 原子核的同位旋 $|I I_3\rangle$

$$I_3^{\text{nucleus}} = \sum I_3^{\text{nucleon}} = \frac{Z-N}{2}.$$

$$\text{Nucleon}: \quad I = 1/2 \qquad \begin{cases} \text{proton}: & I_3 = +1/2 \\ \text{neutron}: & I_3 = -1/2. \end{cases}$$

A=14 同质异位素 （isobars）



I=1同位旋三重态 →

原子核的电磁矩

原子核的磁偶极矩

- 经典物理，环型电流产生磁矩

  μ=I **A** I-环型电流，**A**-电流包围的面积

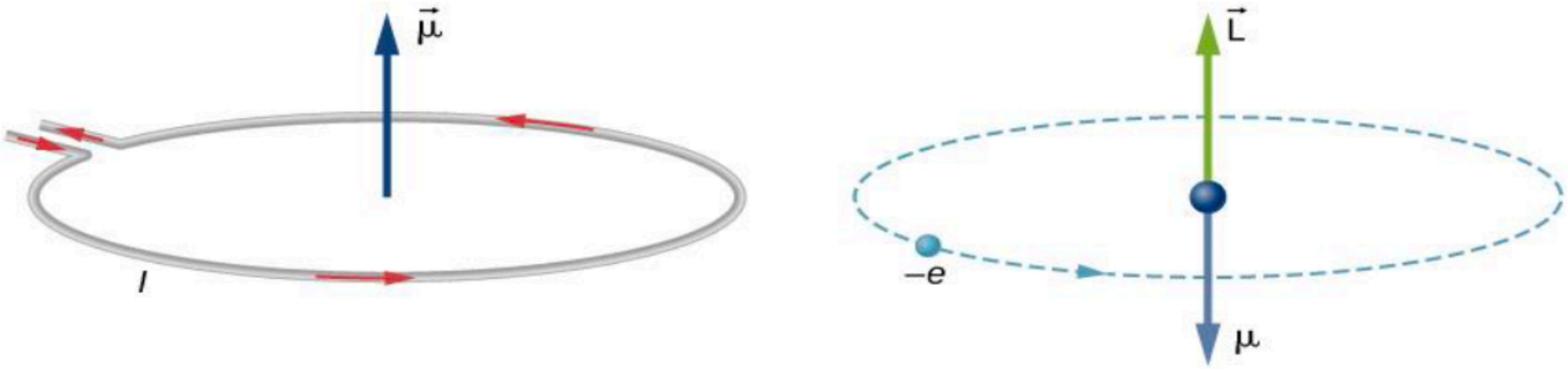
- 考虑电荷q的粒子做圆周运动 (角动量 L = r × p)，电流

$$I = \frac{q}{t} = \frac{q}{2\pi r/v} = \frac{qv}{2\pi r}$$

$$\mu = IA = \frac{qv}{2\pi r}\pi r^2 = \frac{qvr}{2} = \frac{q}{2m}(mvr) = \frac{q}{2m}L$$

- 电子轨道磁矩

  μ = - (e/2m) L



(a) Current-carrying loop (b) Hydrogen atom

原子核的电磁矩

- 原子核的磁偶极矩
  - 电子轨道磁矩

    $\mu = -(e/2m)\,L$

  - 电子自旋磁矩

    $\mu = -(e/2m)\,g\,S$

    In the QED leading order, **$g=2$**

  - 电子反常磁矩（anomalous magnetic moment)

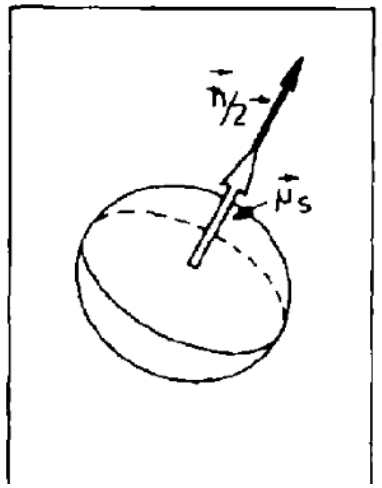
    $a = (g-2)/2$

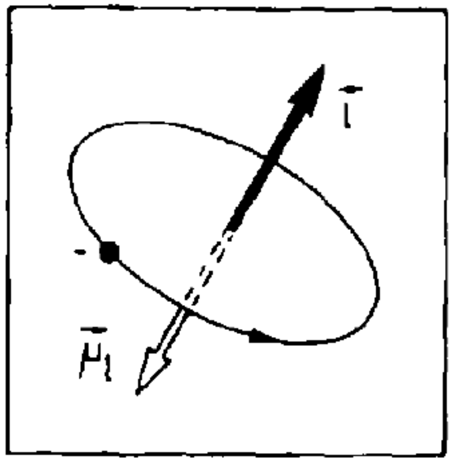
  - QED最大成就： $a_e(SM)=1.15965218188(78)\times10^{-3}$

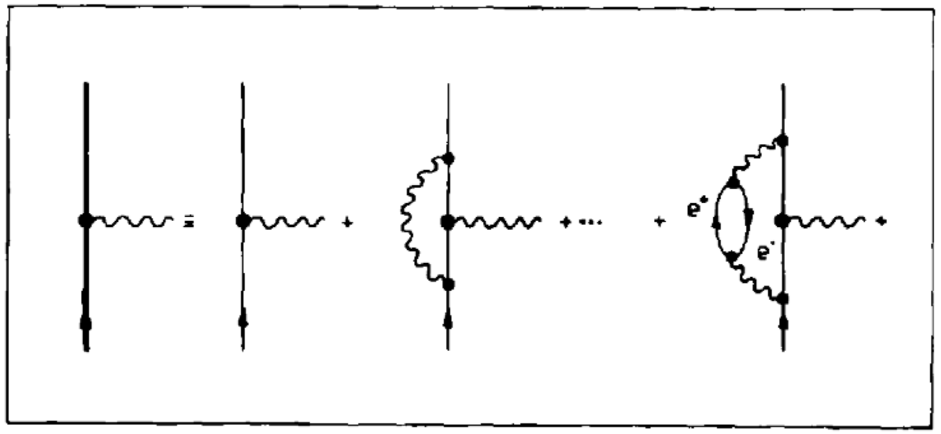
    $a_e(exp)=1.15965218073(28)\times10^{-3}$

  - Muon g-2 $a_\mu(SM)=116591802(2)(42)(26)\times10^{-11}$

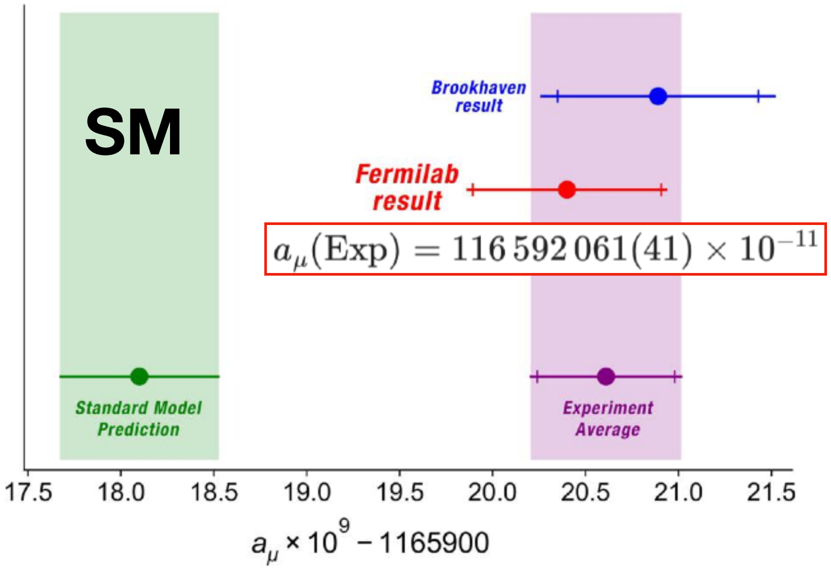
    $a_\mu(exp)=11659208.9(5.4)(3.3)\times10^{-10}$







$$a_{e^-}^{\mathrm{th}} = \frac{1}{2}\left(\frac{\alpha}{\pi}\right) - 0.328479\left(\frac{\alpha}{\pi}\right)^2 + 1.29\left(\frac{\alpha}{\pi}\right)^3,$$



B. Abi *et al.* (Muon $g-2$ Collaboration)
Phys. Rev. Lett. **126**, 141801 – Published 7 April 2021

尧江明

原子核的电磁矩

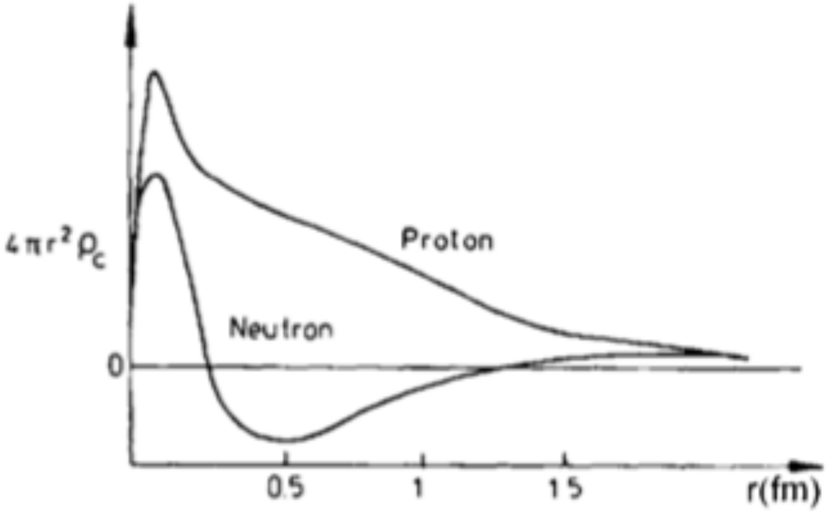
- 原子核的磁偶极矩

电子磁矩 $\vec{\mu}_e = -\frac{e\hbar}{2m_ec}(g_{e,l}\vec{L}+g_{e,s}\vec{S}) \approx -\mu_B(\vec{L}+2\vec{S})$

$g_{e,s}-2\neq 0$ 点粒子

核磁矩 $\vec{\mu}_p = \frac{e\hbar}{2m_pc}(\vec{L}+g_{p,s}\vec{S}) = \mu_N(\vec{L}+g_{p,s}\vec{S})$

$g_{p,s}=5.58, \quad g_{n,s}=-3.82$ 内部结构



对比电子与核子磁矩大小：

$\mu_B = \frac{e\hbar}{2m_ec} = 5.7883826\times10^{-5}\ eV/T$ 玻尔磁子 Bohr magneton

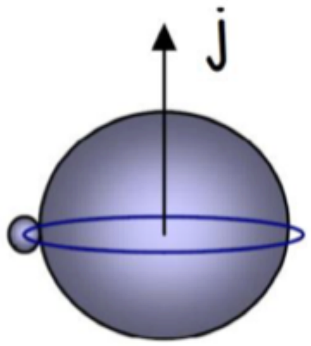
$\mu_N = \frac{e\hbar}{2m_pc} = 3.15245\times10^{-8}\ eV/T$ 核磁子 nucleon magneton

$$\mu_B \approx 10^3\mu_N$$

原子核的电磁矩

- 奇A原子核的磁偶极矩

（假设）基态自旋由未配对核子决定



$$\boxed{\langle jm|\mathbf{A}|jm'\rangle = \langle jm|\mathbf{j}|jm'\rangle \frac{\langle jj|\mathbf{A}\mathbf{j}|jj\rangle}{j(j+1)},}$$

$$\mu = \mu_N \frac{1}{j+1} \langle jj|\, g^l \mathbf{l}\mathbf{j} + g^s \mathbf{s}\mathbf{j} |jj\rangle$$

$$= \mu_N \frac{1}{2(j+1)} \left[g^l\left(j(j+1) + l(l+1) - \tfrac{3}{4}\right) + g^s\left(j(j+1) + \tfrac{3}{4} - l(l+1)\right)\right].$$

For $j = l \pm \frac{1}{2}$ we get

Schmidt 公式

$$\mu = \mu_N \begin{Bmatrix} g^l\left(j - \frac{1}{2}\right) + \frac{1}{2} g^s \\ \left[g^l\left(j + \frac{3}{2}\right) - \frac{1}{2} g^s \right] \dfrac{j}{j+1} \end{Bmatrix} \quad \text{for} \quad \begin{cases} j = l + \frac{1}{2} \\ j = l - \frac{1}{2} \end{cases}.$$

Ring & Schuck, 1980

原子核的电磁矩

- 奇A原子核的磁偶极矩

Schmidt 公式

For $j=l\pm\frac{1}{2}$ we get

$$\mu=\mu_N\left\{\begin{matrix} g^l\left(j-\frac{1}{2}\right)+\frac{1}{2}g^s \\ \left[g^l\left(j+\frac{3}{2}\right)-\frac{1}{2}g^s\right]\dfrac{j}{j+1}\end{matrix}\right\}\quad\text{for}\quad\left\{\begin{matrix} j=l+\frac{1}{2} \\ j=l-\frac{1}{2}\end{matrix}\right..$$

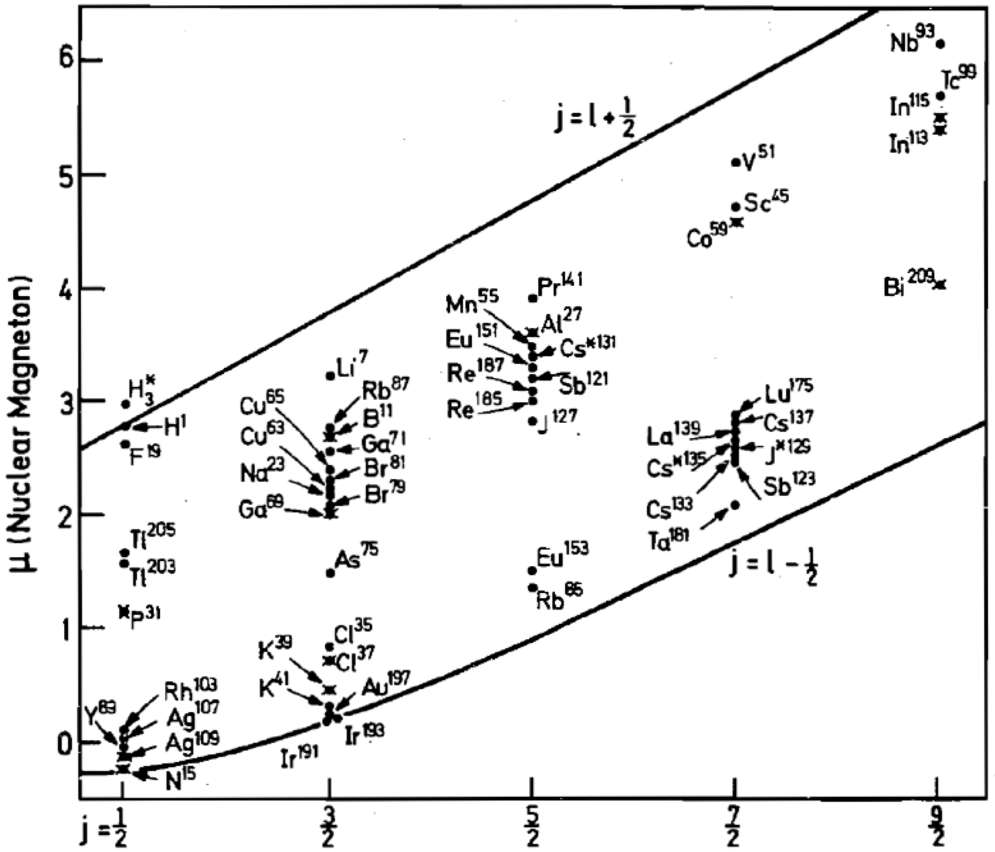


Figure 2.16. Magnetic moments for Z-odd nuclei as a function of angular momentum. (From [MJ 55].)

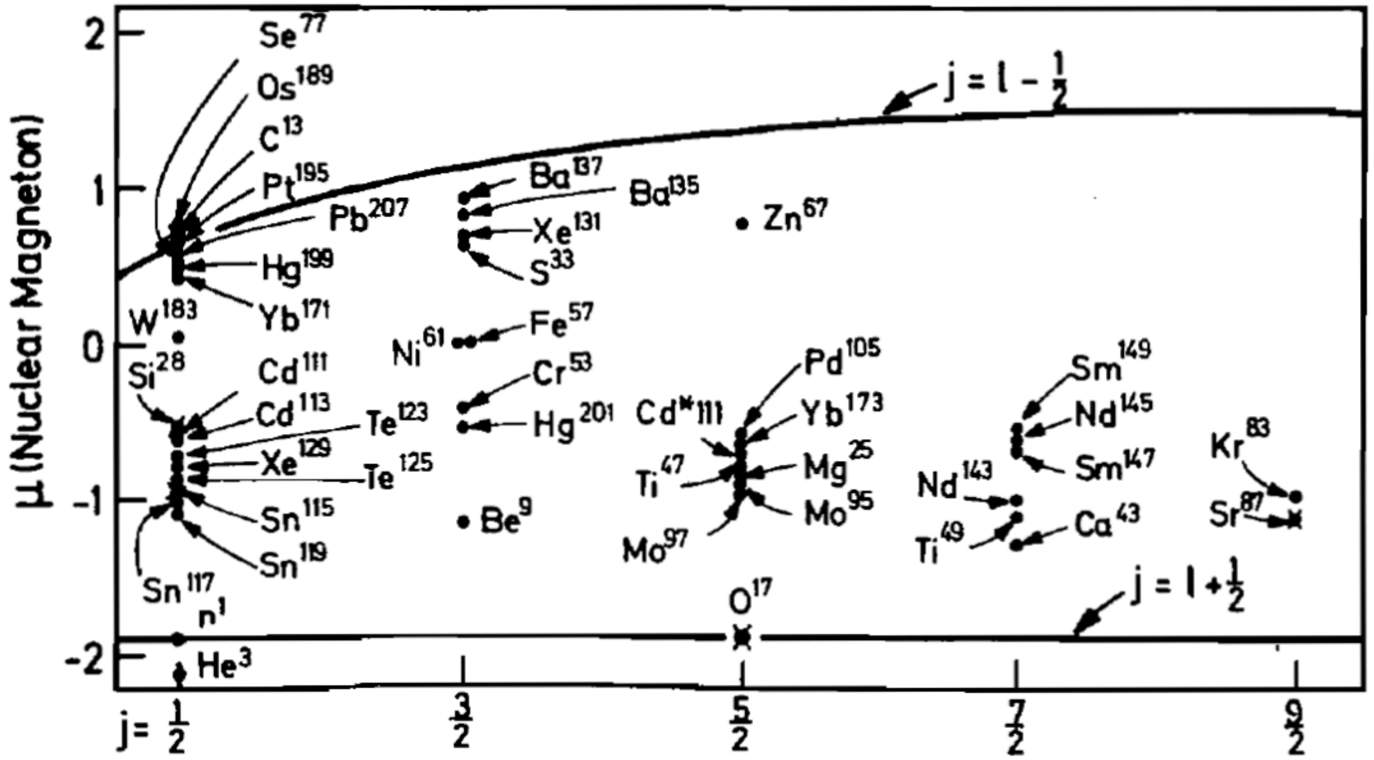


Figure 2.17. Magnetic moments of N-odd nuclei as a function of angular momentum. (From [MJ 55].)

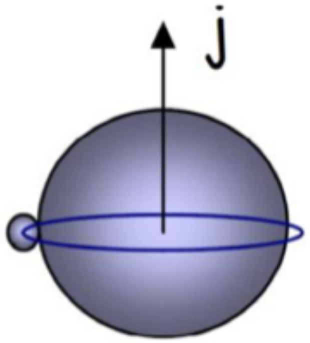
Ring & Schuck, 1980

实验数据与Schmidt 公式偏差：**极端单粒子模型近似缺陷**

原子核的电磁矩

Ring & Schuck, 1980

- 奇A原子核的电四极矩

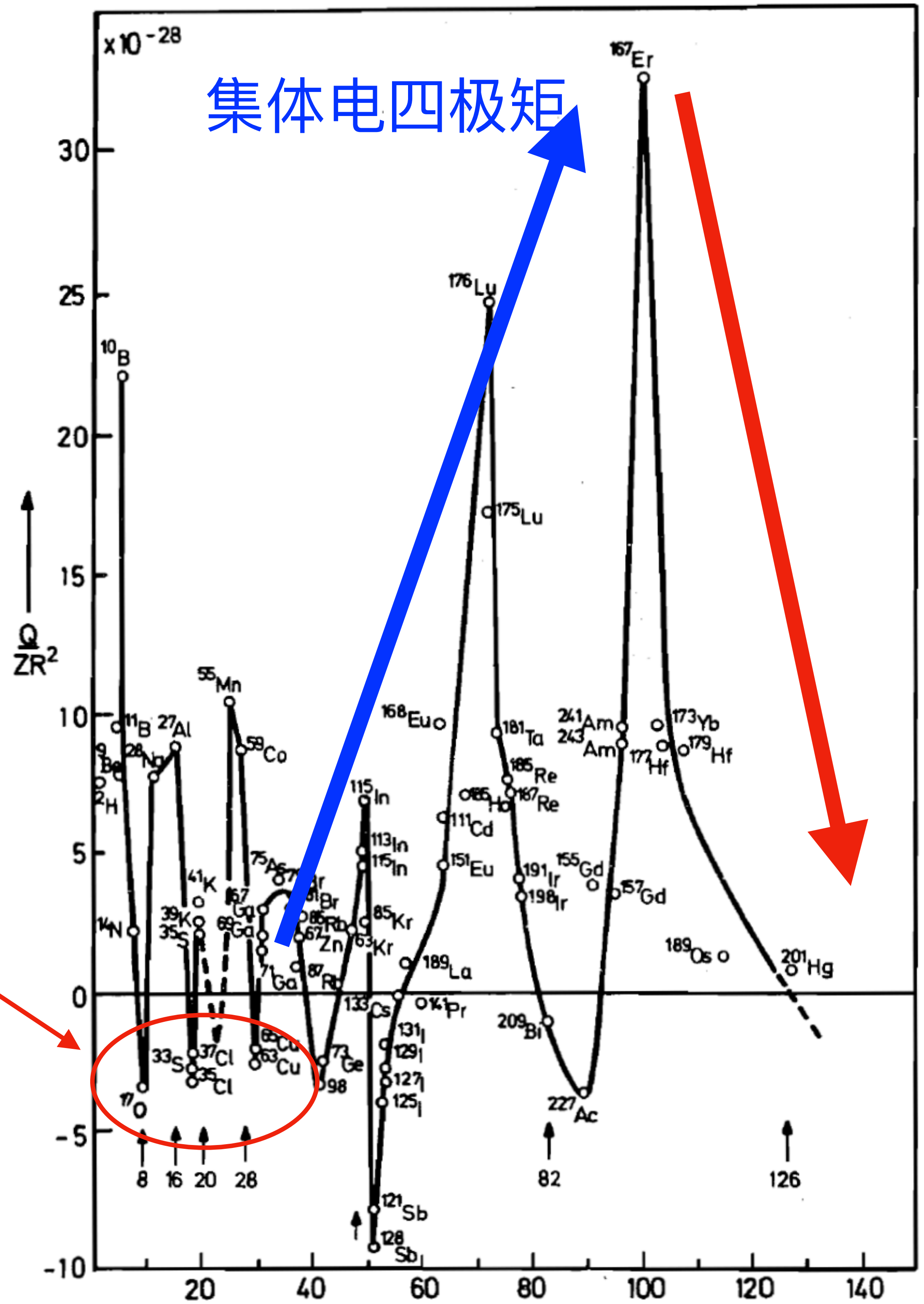


（假设）基态电四极矩完全由未配对核子贡献

$$Q = e\sqrt{\frac{16\pi}{5}}\langle nljj|r^2Y_{20}|nljj\rangle = -e\langle r^2\rangle\frac{2j-1}{2j+2},$$

价核子电四极矩 **Q<0 for j>1/2**

where $\langle r^2\rangle$ is the average square radius of a particle in the state $|nljm\rangle$. For $j>1/2$, the quadrupole moments turn out to be negative, that is, the probability distribution looks like a pancake in the plane perpendicular to the z-axis. For holes we have the opposite (i.e., prolate probability distributions). In Fig. 2.18, we see that this picture is qualitatively right in the neighborhood of magic numbers. However, as one fills the next shell with more and more nucleons, we soon find experimentally a transition to positive quadrupole moments with very large values. We shall see in the next chapter that in this case the average field of the nucleons is no longer spherical and we obtain a deformed density distribution. Only at the end of the shell, when nearly all levels are occupied, does one again get the picture of one or a few holes in a spherical core.

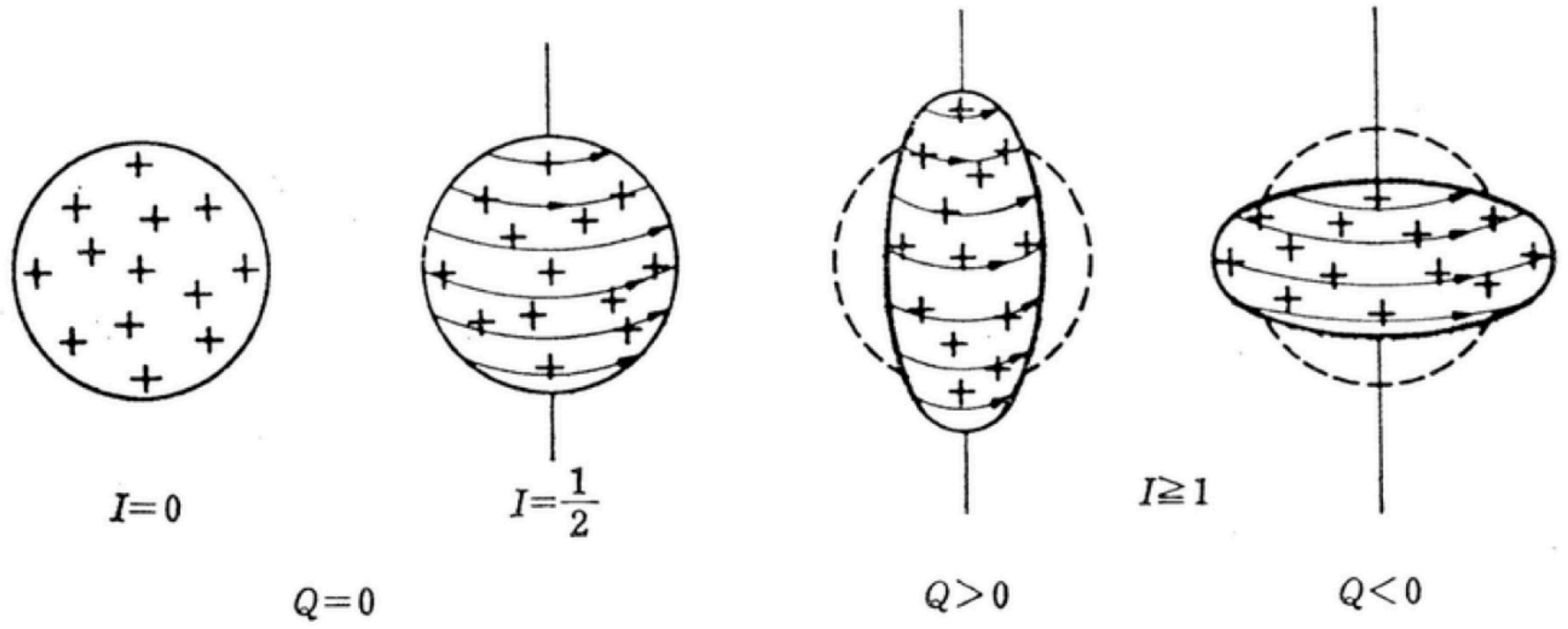


集体电四极矩

Figure 2.18. Quadrupole moments for odd-*A* nuclei against the number of odd nucleons. Arrows indicate closed shells. (From [Se 64].)

原子核的电磁矩

- 原子核的电四极矩
  - **液滴模型/壳模型：大部分原子核接近球形**
  - **形变：轴对称椭球→电四极矩**

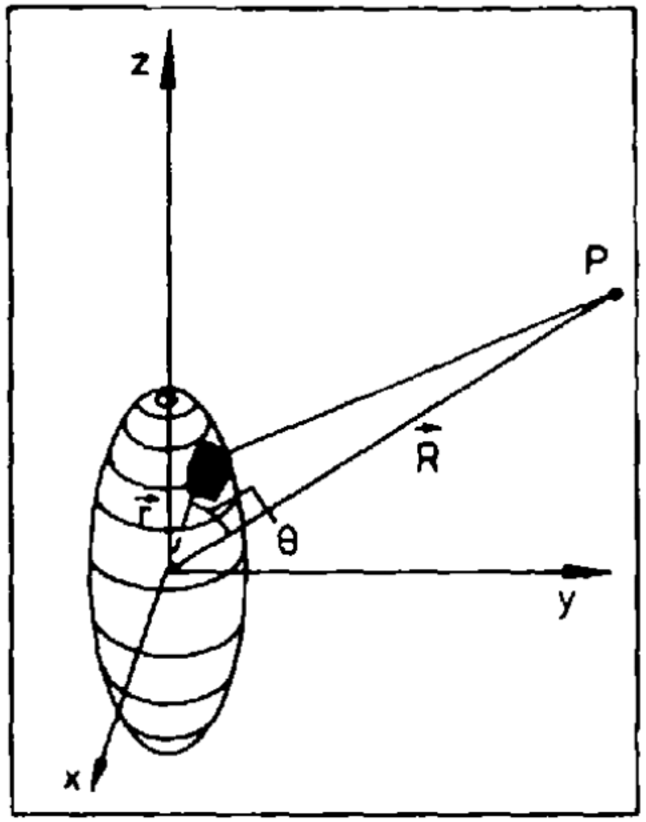


原子核的电磁矩

- 原子核的电多极展开

$$\Phi(\vec{R}) = \frac{1}{4\pi\epsilon_0}\int_{\mathrm{Vol}} \frac{\rho(\vec{r})}{|\vec{R}-\vec{r}|}\mathrm{d}\vec{r},$$

in a series for (r/R). **展开**



$$\Phi(\vec{R}) = \frac{1}{4\pi\epsilon_0}\frac{q}{R} + \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r})r\cos\theta\,\mathrm{d}\vec{r}}{R^2} + \frac{1}{2}\cdot\frac{1}{4\pi\epsilon_0}\int\frac{\rho(\vec{r})(3\cos^2\theta-1)r^2\mathrm{d}\vec{r}}{R^3} + \cdots$$

$$r\cos\theta = \vec{r}\cdot\vec{R}/R = \sum_i x_i X_i/R,$$

$$= \frac{1}{4\pi\epsilon_0}q/R + \sum_i \frac{p_i}{4\pi\epsilon_0}\frac{X_i}{R^3} + \sum_{ij}\frac{1}{2}\frac{1}{4\pi\epsilon_0}\frac{Q_{ij}}{R^5}X_iX_j + \cdots$$

电偶极矩 $$p_i = \int\rho(\vec{r})x_i\mathrm{d}\vec{r},$$

电四极矩 $$Q_{ij} = \int\rho(\vec{r})(3x_ix_j - r^2\delta_{ij})\mathrm{d}\vec{r}.$$

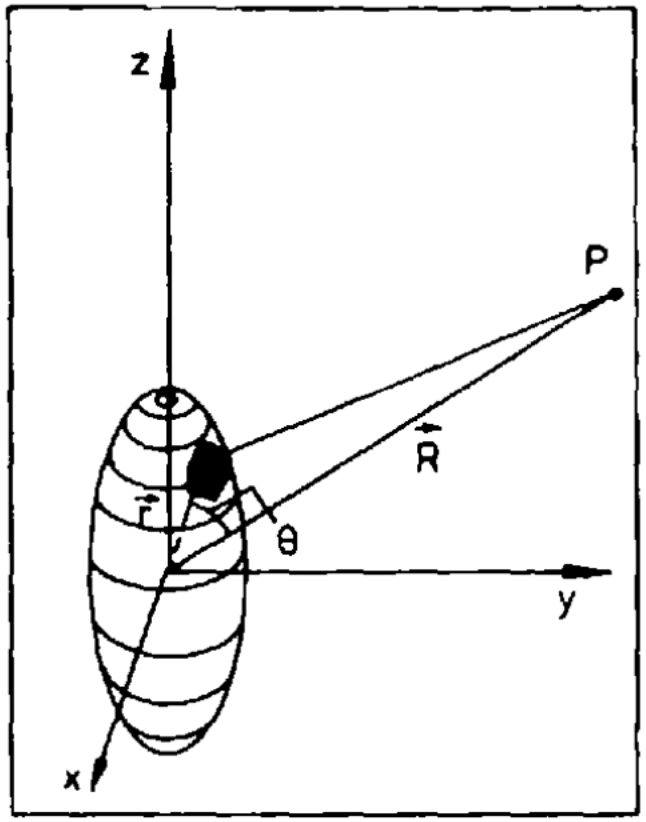
原子核的电磁矩

- 原子核的电四极矩

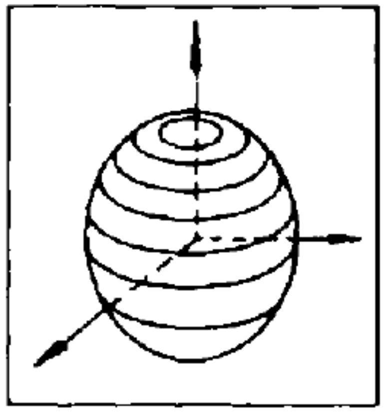
$$Q(J,M)=\int \psi_J^{*M}(\vec{r}_i)\sum_i\left(3z_i^2-r_i^2\right)\psi_J^M(\vec{r}_i)\,\mathrm{d}\vec{r}_i,$$

$$Q_{\mathrm{lab}}=\frac{1}{2}\left(3\cos^2\beta-1\right)Q_{\mathrm{intr}},$$

$$\boxed{\int\rho(\vec{r})\left(3z^2-r^2\right)\mathrm{d}\vec{r},}$$

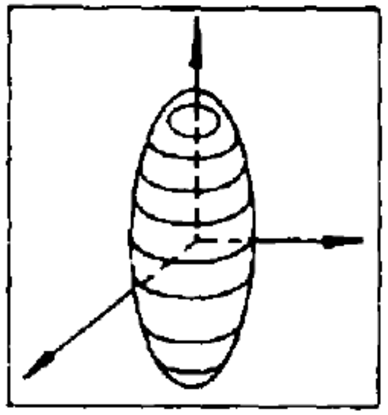


$$r\cos\theta=\vec{r}\cdot\vec{R}/R=\sum_i x_iX_i/R,$$



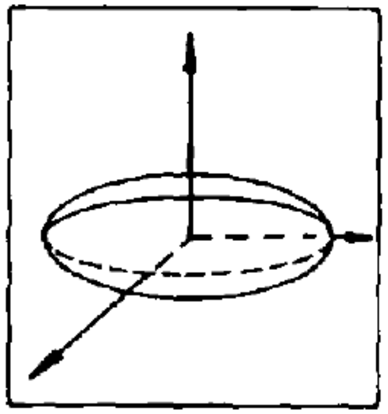
Q=0

球形



Q>0

长椭球



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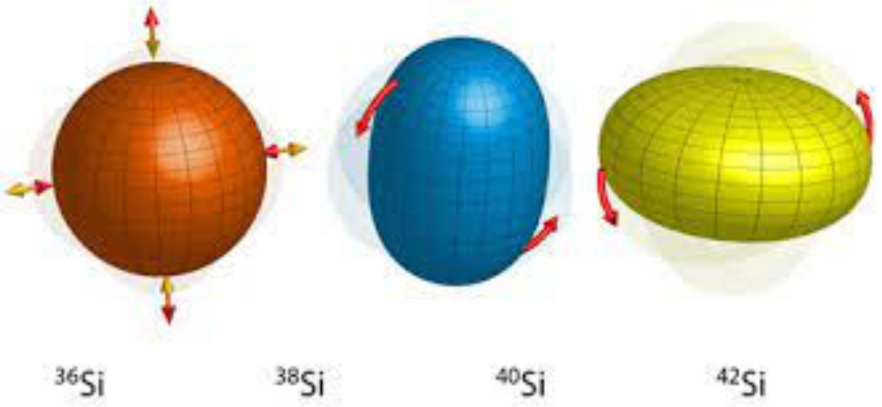

小结

✦ 主要内容

- 结合能、液滴模型、基态量子数、电偶极矩以及电四极矩

✦ 基本概念

- 结合能、分离能、自旋宇称、同位旋、磁矩、电四极矩

✦ 重点、难点

- 液滴模型各项的物理意义

The End