Neutrinoless double beta decay in the in-medium GCM

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DBD-Collaboration Spring Meeting, May 28, 2020

Outline

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3. Nuclear matrix elements of $0\nu\beta\beta$ decay from the IM-GCM calculation
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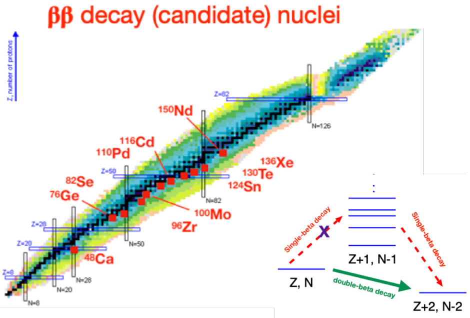
Brief introduction

Candidate $0\nu\beta\beta$ decay in atomic nuclei

- Single-beta decay is energetically forbidden
- Experimental interest
  1. Large $Q_{\beta\beta}$ value
  2. Large isotopic abundance
  3. Low background in the energy region of interest



Nuclear matrix element and effective neutrino mass

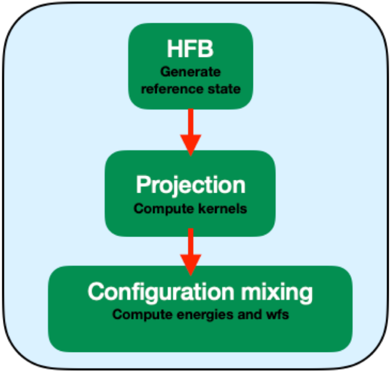
The inverse of half-life of $0\nu\beta\beta$ (exchange light Majorana neutrino) can be factorized as

$$[T^{0\nu}_{1/2}]^{-1} = g_A^4 G_{0\nu} \left|\frac{\langle m_{\beta\beta}\rangle}{m_e}\right|^2 \left|M^{0\nu}\right|^2, \quad |\langle m_{\beta\beta}\rangle| = |\sum_{i=1,2,3} U^2_{ei} m_i|$$

The candidate nuclei are mostly medium-mass open-shell (deformed) nuclei, challenge for *ab initio* methods.

The in-medium generator coordinate method (GCM)

Generator coordinate method (GCM) in a nutshell



- The trial wave function of a GCM state

$$|\Phi^{JNZ\cdots}\rangle = \sum_Q F_Q^{JNZ} \hat{P}^J \hat{P}^N \hat{P}^Z \cdots |\Phi_Q\rangle$$

$|\Phi_Q\rangle$ are a set of HFB wave functions with Q being the so-called generator coordinate (deformation, pairing, cranking...).

- The mixing weight F_Q^{JNZ} is determined from the Hill-Wheeler-Griffin equation:

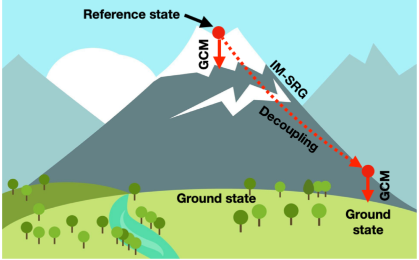
$$\sum_{Q'} \left[H^{JNZ}(Q, Q') - E^J N^{JNZ}(Q, Q') \right] F_{Q'}^{JNZ} = 0$$

Features (pros) of GCM

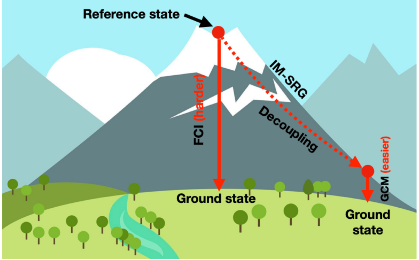
- The Hilbert space in which the H will be diagonalized is defined by the Q. Many-body correlations are controlled by the Q
- The Q is chosen as (collective) degrees of freedom relevant to the physics.
- Dimension of the space in GCM is generally much smaller than full CI calculations.

When GCM meets chiral interactions, ...

- The GCM has been frequently implemented into nuclear DFT calculations.
- How to perform GCM calculations based on Hamiltonians derived from chiral effective field theory?
- We adopt a two-step scheme: IM-GCM (IM-SRG+GCM)



GCM vs IM-GCM based on nuclear chiral interactions



Ab initio many-body approaches for nuclear ground state

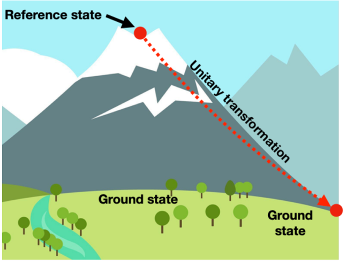
The method: from the reference state to exact solution

- Supposing $|\Psi^{JNZ}\rangle$ is the exact w.f. of (ground) state ($J^\pi = 0^+$), the energy of the state can be calculated as

$$
\begin{aligned}
E_0 &= \langle \Psi^{JNZ}|H_0|\Psi^{JNZ}\rangle \\
&= \langle \Psi^{JNZ}|U^\dagger(s)U(s)H_0U^\dagger(s)U(s)|\Psi^{JNZ}\rangle \\
&\equiv \langle \Phi^{JNZ}|H(s)|\Phi^{JNZ}\rangle
\end{aligned}
$$

where s is an energy scale parameter, and $U(s)$ is a unitary transformation

$$
\begin{aligned}
H(s) &= U(s)H_0U^\dagger(s), \\
|\Phi^{JNZ}\rangle &= U(s)|\Psi^{JNZ}\rangle
\end{aligned}
$$



From a reference state to ground/excited state

$|\Phi^{JNZ}\rangle$ is the wave function of a pre-chosen reference state with less correlation than the ground state.

The method: from the reference state to exact solution

- Supposing the reference state $|\Phi^{JNZ}\rangle$ is not orthogonal to the exact w.f. , one has

$$|\Psi^{JNZ}\rangle = U^{\dagger}(s_{\infty})|\Phi^{JNZ}\rangle.$$

where the $U^{\dagger}(s_{\infty})$ should fulfill the following condition ($n \geq 1$):

$$\langle \Psi^{p_1\cdots p_n}_{h_1\cdots h_n}|H_0|\Psi\rangle = 0, \quad \langle \Psi|H_0|\Psi^{p_1\cdots p_n}_{h_1\cdots h_n}\rangle = 0$$

which is equivalent to

$$\langle \Phi^{p'_1\cdots p'_n}_{h'_1\cdots h'_n}|H(s_{\infty})|\Phi\rangle = 0, \quad \langle \Phi|H(s_{\infty})|\Phi^{p'_1\cdots p'_n}_{h'_1\cdots h'_n}\rangle = 0$$

It indicates that one can get an exact solution of a given Hamiltonian H_0 at hard-energy scale by solving the Hamiltonian $H(s_{\infty})$ evolved down to a soft-energy scale starting from a properly-chosen reference state $|\Phi\rangle$.

IMSRG provides a tool of choice to derive the $U(s)$.

The method: basic idea of IMSRG

- A set of continuous unitary transformations onto the Hamiltonian

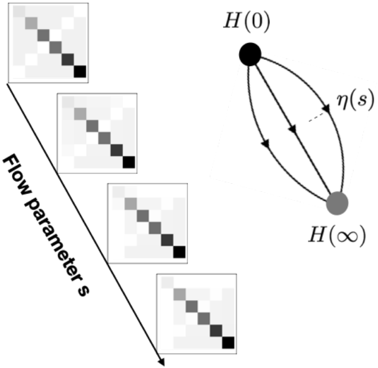
$$H(s) = U(s) H_0 U^\dagger(s)$$

- Flow equation for the Hamiltonian

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

where the $\eta(s) = \frac{dU(s)}{ds} U^\dagger(s)$ is the so-called generator chosen to decouple a given reference state from its excitations.

- Computation complexity scales polynomially with nuclear size

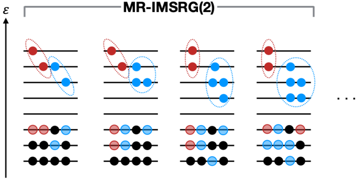


Tsukiyama, Bogner, and Schwenk (2011)
Hergert, Bogner, Morris, Schwenk, Tsukiyama (2016)

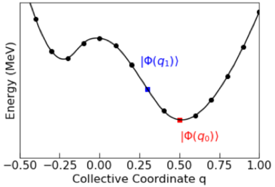
Not necessary to construct the H matrix elements in many-body basis !

Computation of NMEs of $0\nu\beta\beta$: challenges

- open-shell nuclei with collective correlations: mp-mh excitation configurations



MR-IMSRG: build correlations on top of **already correlated** state (e.g., from a method that describes static correlation well)



Thouless Theorem

Starting with a general product wave function $|\Phi_0\rangle$ which is the vacuum to quasi-particle operators β, any other general product wave function $|\Phi_1\rangle$ which is not orthogonal to $|\Phi_0\rangle$ may be expressed in the form

$$|\Phi_1\rangle = \mathfrak{N} \cdot \exp\Big\{ \sum_{k<k'} Z_{kk'} \beta_k^+ \beta_{k'}^+ \Big\} |\Phi_0\rangle,$$

- different unitary transformation for the initial and final nuclei: $U_I(s) \neq U_F(s)$. Computation of the following matrix element

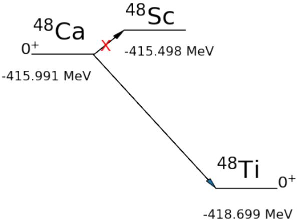
$$M^{0\nu} = \langle \Phi_F | U_F(s) O^{0\nu} U_I^\dagger(s) | \Phi_I \rangle = \langle \Phi_F | e^{\Omega_F(s)} O^{0\nu} e^{-\Omega_I(s)} | \Phi_I \rangle$$

with truncation error controllable is challenge.

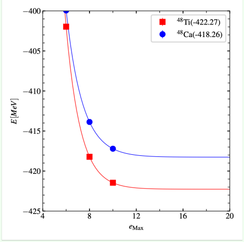
choose the reference state $|\Phi\rangle$ as an ensemble of the initial and final nuclei

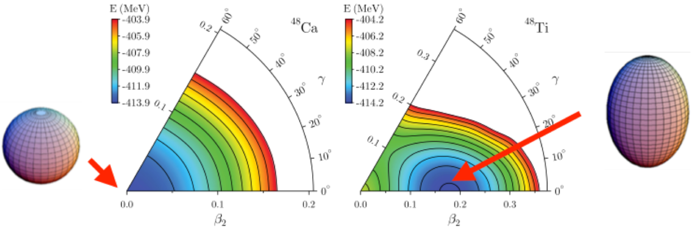
Nuclear matrix elements of $0\nu\beta\beta$ decay from the IM-GCM calculation

^{48}Ca-^{48}Ti: energies

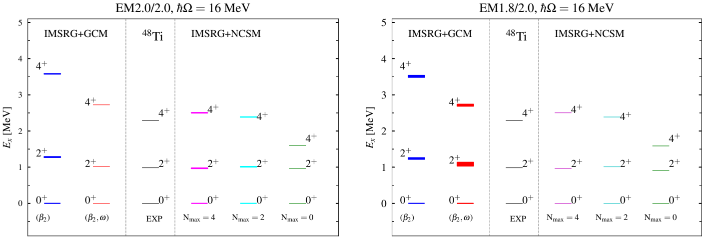


Ground-state energy



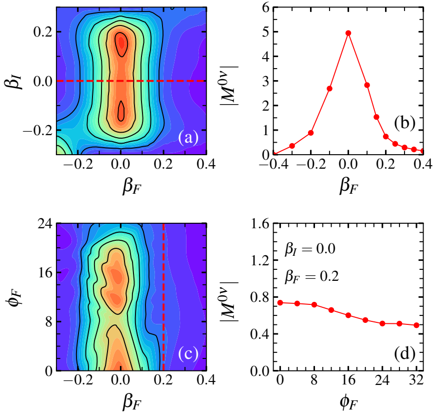


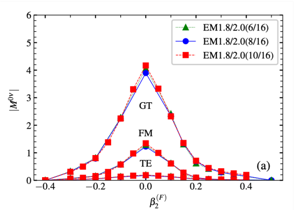
^{48}Ca-^{48}Ti: low-lying spectra

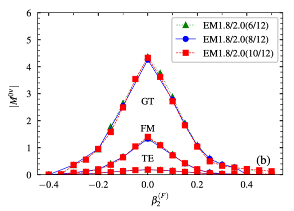


- Spectra by GCM and NCSM based on the same interaction are consistent.
- Low-lying states are reasonably reproduced.
- Inclusion of cranking configurations in GCM calculation compresses the spectrum.

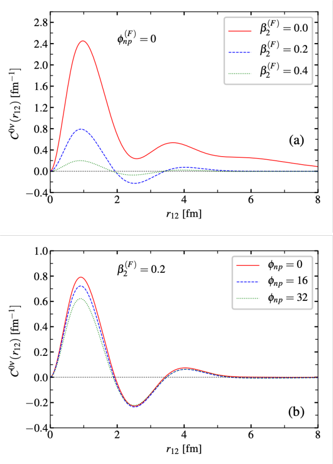
^{48}Ca-^{48}Ti: configuration-dependent NME







^{48}Ca-^{48}Ti: coordinate dependence of the NME

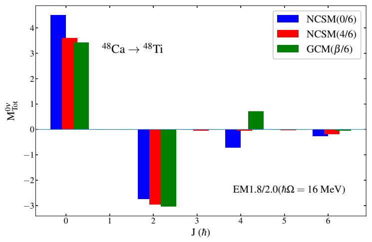


$$M^{0\nu} = \int dr_{12}\, C^{0\nu}(r_{12})$$

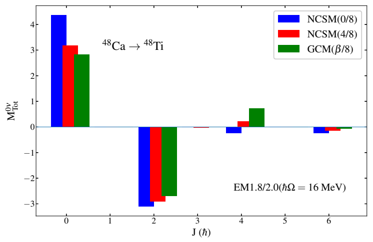
- The quadrupole deformation in ^{48}Ti changes both the short and long-range behaviors
- Neutron-proton isoscalar pairing is mainly a short-range effect

^{48}Ca-^{48}Ti: *J*-component of the NME

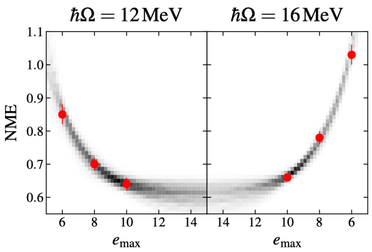
IM-GCM vs IM-NCSM: eMax06

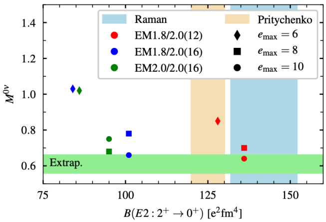


IM-GCM vs IM-NCSM: eMax08



^{48}Ca-^{48}Ti: the final NME



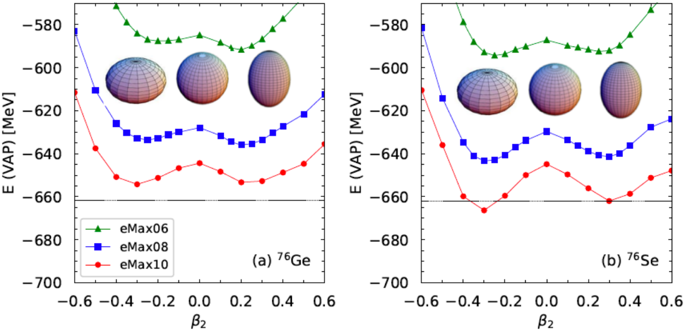


- The value from Markov-chain Monte-Carlo extrapolation is $M^{0\nu} = 0.61^{+0.05}_{-0.04}$
- The neutron-proton isoscalar pairing fluctuation quenches $\sim$17% further, which might be canceled out partially by the isovector pairing fluctuation.

JMY, B. Bally, J. Engel, R. Wirth, T. R. Rodríguez, H. Hergert, PRL (in press)

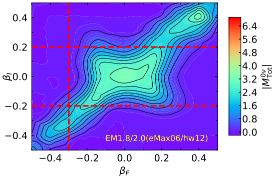
Preliminary results on ^{76}Ge-^{76}Se: PES

- Potential energy surface from the variation after particle-number projection (PNVAP) calculation

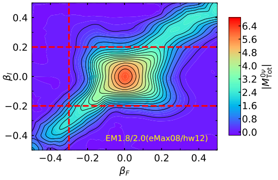


Preliminary results on ^{76}Ge-^{76}Se: NME of the $0\nu\beta\beta$ decay

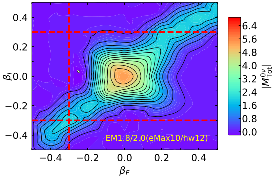
eMax=6

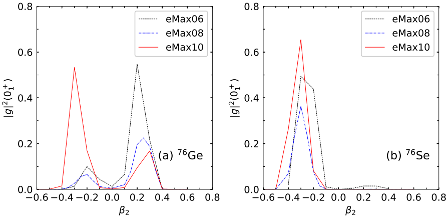


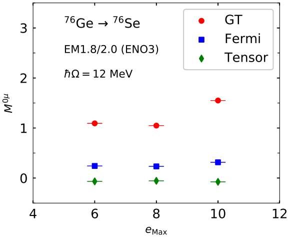
eMax=8



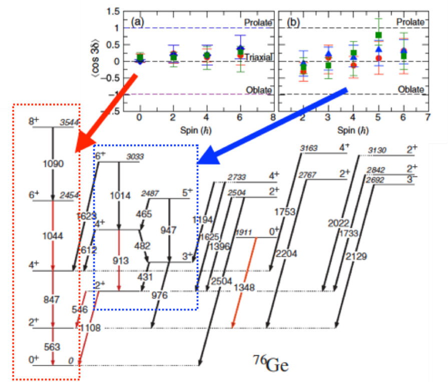
eMax=10



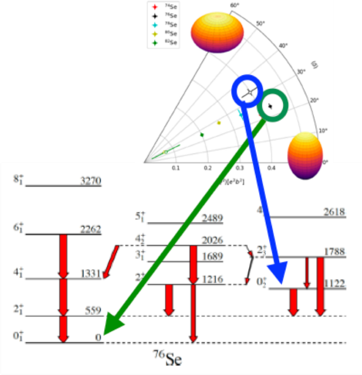




Experimental results on ^{76}Ge-^{76}Se: triaxiality

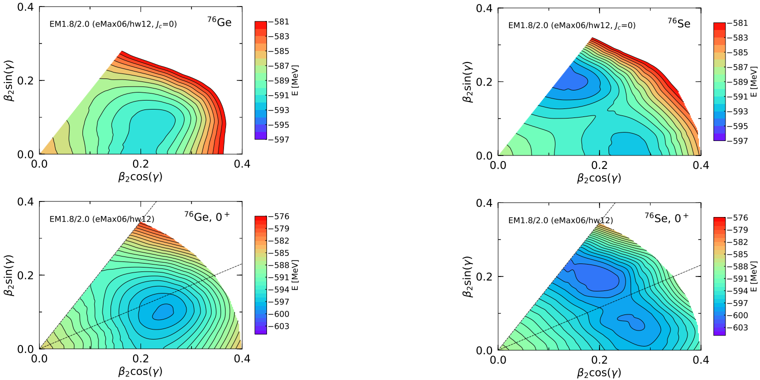


A. D. Ayangeakaa et al., PRL123, 102501 (2019)



J. Henderson et al., PRC99, 054313 (2019)

Preliminary results on ^{76}Ge-^{76}Se: triaxiality effect



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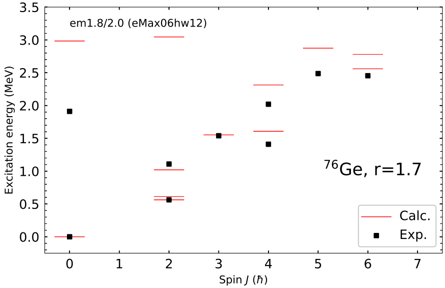
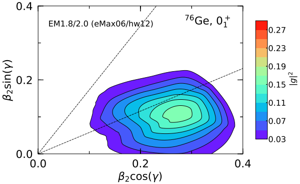
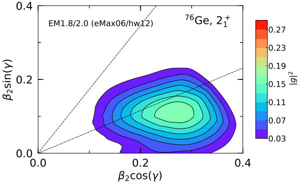
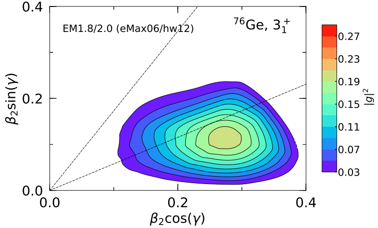
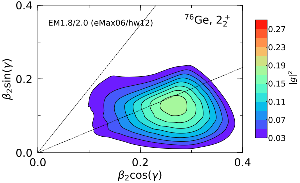


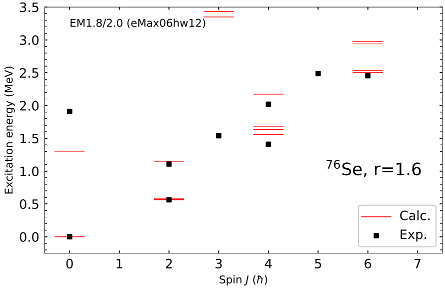


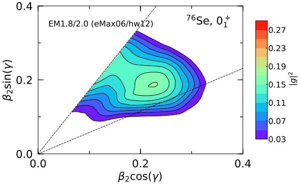


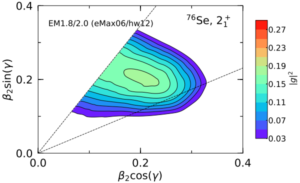


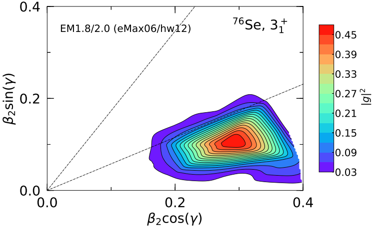


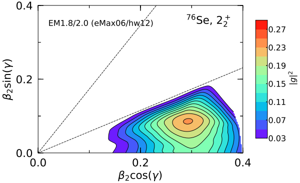
Preliminary results on ^{76}Ge-^{76}Se: triaxiality effect



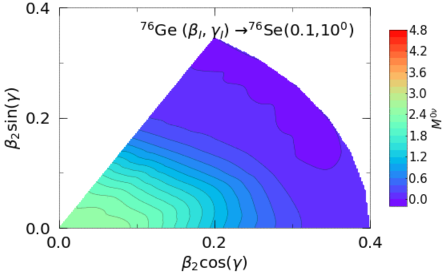


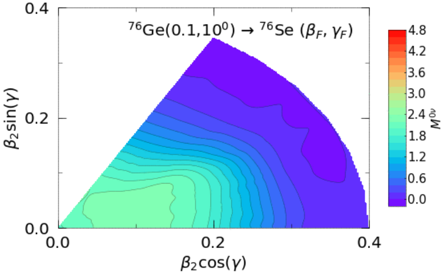




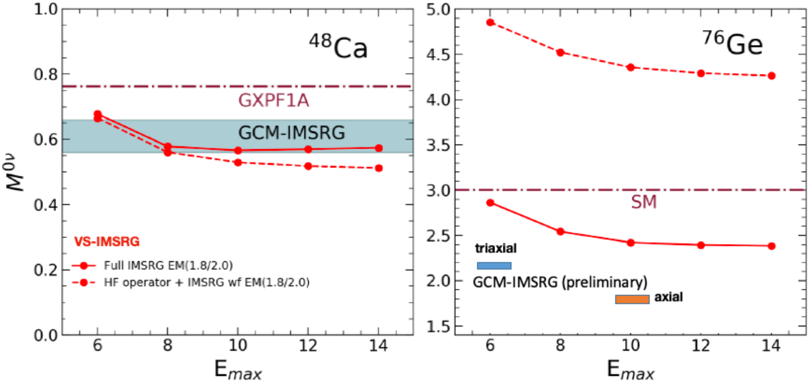


Preliminary results on ^{76}Ge-^{76}Se: triaxiality effect





Preliminary results on ^{76}Ge-^{76}Se: comparison with VS-IMSRG



- The renormalization effect on ^{48}Ca is opposite to that on ^{76}Ge, which is consistent with our finding in IM-GCM calculation. (Please ask me why?)

Note: Figure with VS-IMSRG results is taken from A. Belley, R. Stroberg, J. D. Holt, etc

Summary and outlook

Summary and outlook

Summary

- The nuclear matrix elements (NMEs) are required to determine the neutrino effective mass from $0\nu\beta\beta$-decay experiments. Most of the candidate nuclei are medium-mass open-shell nuclei which are challenge for nuclear ab initio methods.
- We have developed a novel multi-reference framework of IM-GCM (IMSRG+GCM) which opens a door to modeling deformed nuclei with realistic nuclear forces.
- With the IM-GCM, we computed the NMEs for candidate process in medium-mass nuclei ^{48}Ca and ^{76}Ge(preliminary) , which in both cases are smaller than the predictions by (most) phenomenological models. The IM-GCM results seem to be consistent with those from the valence-space IMSRG calculations.

Outlook

- Computation with a larger model space
- Uncertainty quantification, two-body transition currents, 3B operators, etc.
- More benchmarks against other *ab initio* calculations